Name: ______________________ PennID: ______________

Math 240 Final Exam
December 14, 2017

Encircle your Professor, TA and recitation time below:

Professor	Simmons	Wibmer	Zhu	
Nibras Islam	We 8-9	We 9-10	Fr 8-9	Fr 9-10
Patrick Emedom-Nnamdi	We 8-9	We 9-10	Fr 8-9	Fr 9-10
Brian Luo	Mo 8-9	Mo 9-10	We 8-9	We 9-10
Benjamin Foster	Tu 8:30-9:30	Tu 9:30-10:30	Th 8:30-9:30	Th 9:30-10:30

Instructions

Turn off and put away your cell phone. Please write your Name and PennID on the top of this page and encircle your professor, TA and recitation time. Please sign and date the pledge below. No calculators or any other electronic devices are allowed during this exam. You may use a cheat sheet but no other material. Read each question carefully, and answer each question completely. Show all of your work. Write all your solutions clearly and legibly; no credit will be given for illegible solutions. Please try to be as organized as possible and clearly indicate your final solution (in the space provided). If any question is not clear, ask for clarification.

#	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

______________________ ______________

Signature Date

Problem 1 (10 pts): Decide whether the following statements are true or false. You do NOT have to justify your answer. (Please indicate your answer by encircling "true" or "false".)

1. The determinant of a matrix whose entries are all positive must be positive.

   True False

2. If $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a basis for $\mathbb{R}^n$ and A is an invertible $n \times n$-matrix, then $\{A\mathbf{v}_1, \ldots, A\mathbf{v}_n\}$ must also be a basis for $\mathbb{R}^n$.

   True False

3. There is a unique linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ that takes both $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

   True False

4. If L is a linear differential operator and y_1, y_2 are solutions to $Ly = F(x)$, then $y_1 - y_2$ is a solution to $Ly = 0$.

   True False

5. If A^2 is diagonalizable, then also A must be diagonalizable.

   True False

Problem 2 (10 pts): True or False? You need to justify your answer to receive credit!

1. If A and B share a common eigenvector $\mathbf{v}$, then $AB - BA$ is not invertible.

   Circle one: True False

   Justification:

2. A polynomial differential operator of order 5, annihilates every polynomial of degree less than or equal to 4.

   Circle one: True False

   Justification:

Problem 3 (10 pts): Compute the reduced row-echelon form, the rank and a basis for the nullspace for the following matrix A:

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

RREF(A) =________________________________ rank(A) =____ basis:________________________________

Problem 4 (10 pts): If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $\det(A) = 5$, what is

$$\begin{vmatrix} 2a & d+3a & g \\ 2b & e+3b & h \\ 2c & f+3c & i \end{vmatrix}?$$

$\det(A) =$__________

Problem 5 (10 pts): Let S be the subset of $M_{2\times 2}(\mathbb{R})$ consisting of all 2×2-matrices whose entries add up to zero. Show that S is a subspace of $M_{2\times 2}(\mathbb{R})$.

Problem 6 (10 pts):

1. Let $T : \mathbb{R}^3 \longrightarrow P_3(\mathbb{R})$ be the linear transformation defined by

$$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = 2a - (a+b-c)x + (2c-a)x^3.$$

Compute $[T]_B^C$ relative to the standard bases $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ and $C = \{1, x, x^2, x^3\}$.

$[T]_B^C =$____________________________________

2. Compute $[T(\mathbf{v})]_C$ for $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$.

$[T(\mathbf{v})]_C =$________________

Problem 7 (10 pts): Decide whether or not the matrix A is diagonalizable. If so, find an invertible matrix S and a diagonal matrix D such that $S^{-1}AS = D$.

$$A = \begin{bmatrix} -4 & -3 & 3 \\ 6 & 5 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

(**Hint**: $\lambda = 2$ is an eigenvalue of A.)

Diagonalizable? Yes No If yes, $S =$____________________ $D =$____________________

Problem 8 (10 pts): Suppose A is a 4×4 defective matrix with characteristic polynomial

$$p(\lambda) = (\lambda + 2)^2(\lambda - 1)(\lambda + 1).$$

1. Find the algebraic multiplicity and geometric multiplicity of -2. Briefly explain your reasoning.

   algebraic multiplicity:____________ geometric multiplicity:____________

2. What is the rank of $A + 2I$? Briefly explain your reasoning.

   $\text{rank}(A + 2I) =$____________

3. Find the rank of A. Briefly explain your reasoning.

   $\text{rank}(A) =$____________

Problem 9 (10 pts): Solve the following initial value problem:

$$y'' + y' - 6y = 10e^{2x}, \quad y(0) = 0, \quad y'(0) = 7.$$

$y(x) =$ ______________________________________

Problem 10 (10 pts): Determine the general solution of $\mathbf{x}' = A\mathbf{x}$ for

$$A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

(**Hint**: $\det(A - \lambda I) = -(\lambda - 4)^3$.)

$\mathbf{x}(t) =$ ____

Problem 11 (10 pts): Find the general solution to the following linear differential equation, given that x^2 is a solution.

$$x^2y'' - 4xy' + 6y = 0, \quad x > 0.$$

$y(t) =$____

Problem 12 (10 pts): Characterize the equilibrium point of

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix} \mathbf{x}(t)$$

as:

- stable, unstable, neither stable or unstable and
- node, saddle point, proper node, degenerate node, center, spiral point.

You need to justify your answer.

circle one: stable unstable neither

circle one: node saddle point proper node degenerate node center spiral point

Scratch paper

Scratch paper