Name: ______________________________ PennID: ______________

Math 240 Final Exam, December 16th, 2019

Instructions:

- *Turn off and put away your phone. Write your Name (legibly!) and PennID on the top of this page. Sign and date the pledge below. No calculators or any other electronic devices are allowed during this exam. You may NOT consult any material during this exam, other than one double-sided sheet of handwritten notes.*
- *Make sure your exam has* 15 *pages and* 11 *problems. If not, ask for a new exam booklet immediately.*
- *Show all of your work. Write all your solutions clearly.*
- *This test booklet includes three sheets of scratch paper. If you wish to have work on these pages graded, please leave a note on the page containing the question.*
- *If any question is unclear, ask for clarification.*

GOOD LUCK!

#	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
Total	120/120	

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

______________________________ ______________

Signature Date

Problem 1 (10 pts): Consider the 3×3 matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & 0 \\ 0 & -1 & -2 \end{pmatrix}.$$

(a) Find A^{-1}, or explain why it is not possible to do so.

(b) Find all solutions $\mathbf{x}$ to the system of linear equations

$$A\mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

or explain why there are no solutions.

Problem 3 (10 pts): Let A equal the following 7×7 matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}.$$

(a) Compute the Rank and Nullity of the matrix A.

(b) Compute all the eigenvalues of A. *Hint:* $(1,1,1,1,1,1,1)^T$ *is an eigenvector.*

(c) Find a diagonal matrix similar to A.

Problem 5 (10pts): Let V be the subset of space of all 2×2 matrices, $M_2(\mathbb{R})$, that commute with the matrix A:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Is V a subspace of $M_2(\mathbb{R})$? If so, what is the dimension of V?

Problem 7 (10 pts): Let V be the real vector space of functions spanned by e^{2x}, $\sin(x)$, and $\cos(x)$. Let $T : V \to V$ be the linear transformation $T(f(x)) = 3f''(x) - 2f'(x) + 3f(x)$.

(a) Show that T is a linear transformation.

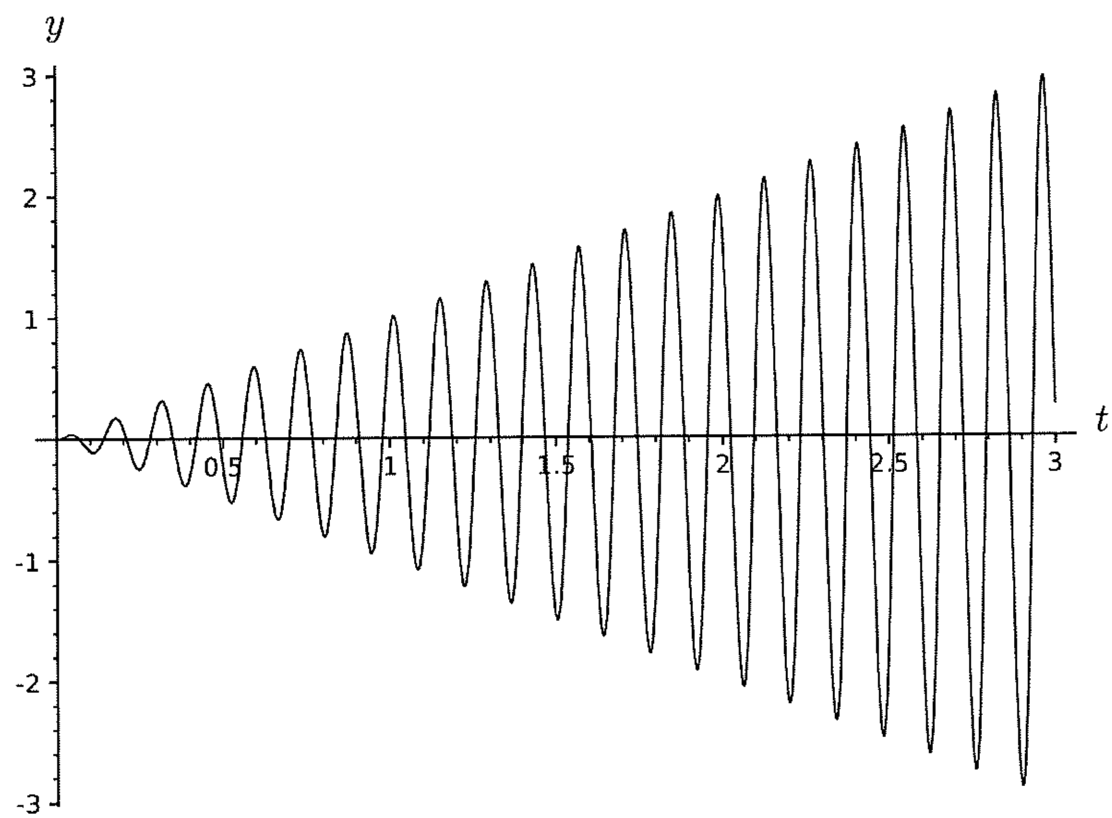
(b) Write down the matrix for T with respect to the ordered basis $\{e^{2x}, \sin(x), \cos(x)\}$.

Problem 9 (10 pts): Find the general solution of the second order differential equation

$$y'' + 2y' + y = e^{-t}\sec^2(t), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

Problem 11 (10 pts): Consider a spring-mass system with mass 0.1 kg, damping constant c and spring constant k. At time $t = 0$, the system is at rest. Suppose that an external force equal to $3\cos(15t)$ is applied to this spring mass system starting at time $t = 0$. The graph of y versus t is given below, where y denotes the displacement of the mass from its equilibrium. In the graph below, the amplitude of oscillation increases without bound as $t \to \infty$.

Solve for the constants c and k.



This page is provided for scratch work. If you wish to have work on this page graded, write "see scratch page 1" on the page containing the original problem.

This page is provided for scratch work. If you wish to have work on this page graded, write "see scratch page 3" on the page containing the original problem.