Final Exam - Math 103 - Fall 2018

This is a 14 question exam. Each problem is worth 10 points. Circle your answers. Show your work - correct answers with little or no supporting work will receive little or no credit. Partial credit may be given for wrong answers if there is significant progress towards a solution.

Name: (please print) ____________________________

Circle the name of your lecturer: Weisshaar Taskovic Rimmer

TA's Name: (please print) ____________________________

Day of week and time of recitation:____________________

Use the space provided to show all work. Two sheets of scrap paper are provided at the end of the exam, do not rip them off. If you write on these scrap pages or the back of any page please and expect that to be graded, indicate this in some strong way. You have 120 minutes to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave. Once you have completed the exam, sign the academic integrity statement below. Do NOT write in the grid below. It is for grading purposes only.

My signature below certifies that I have complied with Penn's Code of Academic Integrity in completing this exam

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Signature

The table below is for grading purposes - do not write below this line

1.________	8.________
2.________	9.________
3.________	10.________
4.________	11.________
5.________	12.________
6.________	13.________
7.________	14.________

Problem 1. Let

$$L = \lim_{x \to 0} \frac{e^x - x - 1}{x^2},$$

and let

$$M = \lim_{x \to 1^-} \frac{1+x}{1-x}.$$

Which of the following statements is true?

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a. $L = +\infty$, $M = +\infty$

b. $L = 1/2$, $M = 1$

c. $L = 1/2$, $M = +\infty$

d. $L = +\infty$, $M = 2$

e. $L = 0$, $M = -\infty$.

Problem 2. What is the distance from the point $(1, 1)$ to the line $y = 3x + 1$?

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a. 0
b. $3/\sqrt{10}$
c. $9/10$
d. 1
3. $\sqrt{5}/2$

Problem 3. Let $f(x) = (2x + 1)^4 - 24x^2 + 5x$. Which of the following statements is true?

a. f has no inflection points.

b. f has exactly 1 inflection point.

c. f has 2 inflection points.

d. f is concave up on the interval $(-1, 0)$.

e. f is concave down on the interval $(2, \infty)$.

Problem 4. Let $f(x) = 3x^{\frac{2}{3}} - x$. Find the x-coordinates of all local and absolute maxima and minima for f. You may assume without justification that $\lim_{x\to-\infty} f(x) = +\infty$ and $\lim_{x\to\infty} f(x) = -\infty$.

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a. f has a local min at $x = 0$ and a local max at $x = 8$, but no absolute maxima or minima.

b. f has a local min at $x = 8$ and a local max at $x = 0$, but no absolute maxima or minima.

c. f has a local and absolute min at $x = 0$ and a local and absolute max at $x = 8$.

d. f has a local and absolute min at $x = -8$ and a local and absolute max at $x = 0$.

e. f has a local and absolute max at $x = 8$, but f has no local or absolute minima.

Problem 5. Suppose

$$f(x) = \begin{cases} \frac{c\sqrt{x}-c}{x-1} & x > 1 \\ x - c & x \leq 1. \end{cases}$$

What value of c makes f continuous everywhere?

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a. $c = 0$
b. $c = 1$
c. $c = 2/3$
d. $c = -1/3$
e. There is no such c.

Problem 6. Using the definition, calculate the derivative $f'(1)$ if

$$f(x) = 2 + \sqrt{1 + 3x}.$$

Problem 7. The curves $y = x^2 + ax + b$ and $y = x^2 - cx + 1$ have a common tangent at the point $(1, 3)$. Find a, b and c.

Problem 8. Find the line that is normal to the curve

$$x \sin 2y = y \cos 2x$$

at the point $(\frac{\pi}{4}, \frac{\pi}{2})$.

Problem 9. Water is flowing at a rate of $150\pi\,\mathrm{m}^3/\mathrm{min}$ from a shallow concrete conical reservoir (vertex down) of base radius 25m and height 5 m.

(a) How fast is the water level falling when the water is 2 m deep?

(b) How fast is the radius of the water's surface changing then?

Problem 10. Use linearization to approximate the value of the function $f(x) = \frac{x}{x+1}$ at $x = 1.2$.

Problem 11. Estimate the area under the graph of $f(x) = \frac{36}{(x+1)^2}$ between $x = 0$ and $x = 4$ using 4 rectangles and the left endpoint method.

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a. $\frac{144}{5}$

b. $\frac{152}{3}$

c. $\frac{131}{2}$

d. $\frac{205}{4}$

e. $\frac{1669}{100}$

Problem 12. Let

$$L = \int_0^4 \frac{36}{(x+1)^2}\,dx,$$

and let

$$M = \int_0^{\ln 6} e^{2x}\,dx.$$

Find $L \cdot M$.

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a. 324

b. 360

c. 424

d. 443

e. 504

Problem 13. Let

$$L = \int_0^{\frac{\pi}{3}} \frac{\sin x}{(\cos x)^5} dx,$$

and let

$$M = \int_1^4 \frac{(\sqrt{x}-1)^2}{\sqrt{x}} dx.$$

Find $L \cdot M$.

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a. $\frac{5}{2}$

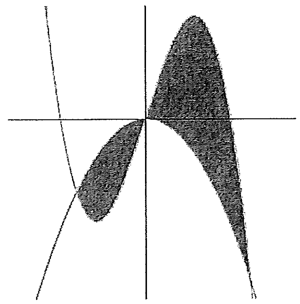
b. $\frac{2}{3}$

c. $\frac{5}{4}$

d. $\frac{15}{2}$

e. $\frac{5}{3}$

Problem 14. Find the shaded area between the curves $y = -x^2$ and $y = -x^3 + 6x$.



a. $\frac{203}{3}$

b. $\frac{501}{4}$

c. $\frac{253}{12}$

d. $\frac{163}{2}$

e. $\frac{121}{5}$