Math 103-002 Rimmer
Final Exam Spring 2018

First and Last Name _____(PRINT) Penn ID_____

Recitation **(circle one):** M 9:00 212 W 8:00 213 W 9:00 214

This exam has 15 questions. Each question is worth 10 points for a total of 150 points. Questions with two parts will be 5 points for each part. Partial credit will be given for the entire exam so be sure to show all work. Circle the correct answer and give supporting work, **a correct answer with little or no supporting work will receive little or no credit.** Use the space provided to show all work. A sheet of scrap paper is provided at the end of the exam, do not rip it off. If you write on the back of any page please indicate this in some way.

You have **120 minutes** to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X5" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave.

Do **NOT** write in the grid below. It is for grading purposes only.

Problem	Points	Problem	Points
1		9	
2		10	
3		11	
4		12	
5		13	
6		14	
7		15	
8			
Total			

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.

_____

Name (printed)

_____ _____

Signature **Date**

1.

a. Simplify as much as possible

$$\frac{1}{\log_2\left(\frac{1}{8}\right)} + e^{2\ln\left(\frac{2}{\sqrt{3}}\right)} + \sec^2\left(\arctan(1)\right)$$

A) 1
B) 2
C) 3
D) 4
E) 6
F) 7
G) 8
H) None of these

b. Solve the equation for x.

$$4x^4 - 5x^2 = -1$$

2. Find the sum $A+B+C$, where

$$A=\lim_{x\to\infty}\arctan\left(\ln\left(x^2+1\right)\right),\qquad B=\lim_{x\to 1}\frac{x^2-1}{3x^2-4x+1},\qquad C=\lim_{x\to\infty}\frac{1}{1+\dfrac{1}{1+\dfrac{1}{x}}}$$

A) $\dfrac{\pi}{2}$

B) $\dfrac{\pi+3}{2}$

C) 1

D) 0

E) $\dfrac{3}{2}$

F) $\dfrac{1}{2}$

G) Does Not Exist

H) None of these

3. Find the constants a and b such that the following function is continuous.

$$f(x)=\begin{cases} 2ax+b & x\le 1 \\ ax^2+2 & 1<x<5 \\ bx^2+27 & x\ge 5 \end{cases}$$

A) 1
B) 2
C) 3
D) 4
E) 6
F) 7
G) 8
H) None of these

Find the value of Find the constants $\frac{a}{b}$.

4. Let

$$f(x)=\frac{-2}{x}$$

Find $f'(x)$ **using the definition of the derivative**.

5. Let

$$f(x)=\frac{x^3-5}{x^2-3}$$

Find $f'(1)$.

A) $\frac{-3}{2}$

B) $\frac{5}{2}$

C) 1

D) 0

E) $\frac{3}{2}$

F) $\frac{1}{2}$

G) Does Not Exist

H) None of these

6. Let

$$f\left(x\right)=8\sin^{3}\left(e^{x}\right).$$

Find $f'\left(\ln\left(\frac{\pi}{3}\right)\right)$

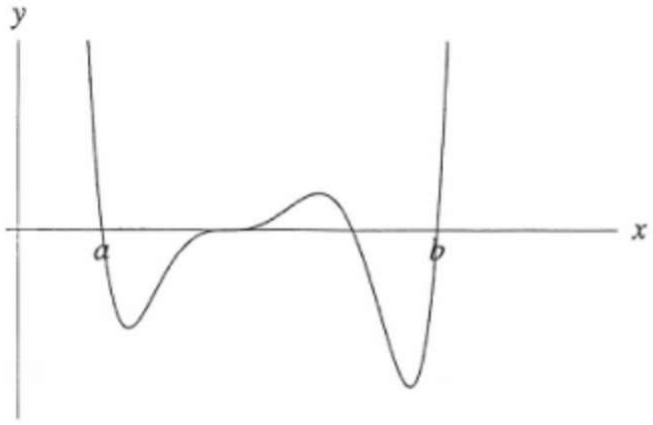
7. Pick any number in the interval $[0,5]$. The value of the function

$$f(x)=\frac{10}{2x^2-8x+9}$$

evaluated at your chosen number is the score you will get for this problem.

8. Below is the graph of a function called

$f(x)$.



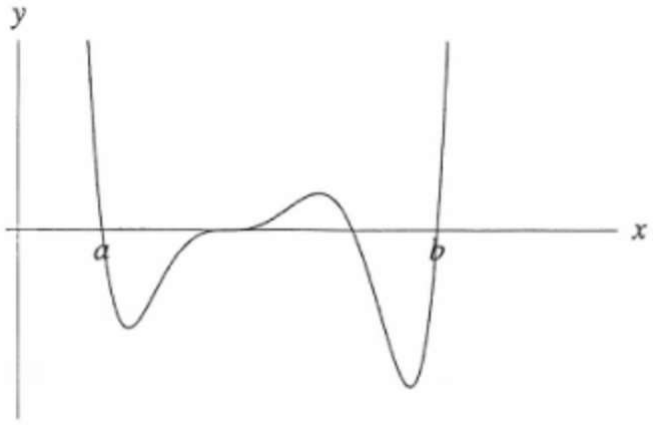
a. On the interval $[a,b]$ how many critical numbers does $f(x)$ have? Explain

b. On the interval $[a,b]$ how many inflection points does $f(x)$ have? Explain

9. Unrelated to the previous question

Below is the graph of a **derivative** called

$g'(x)$.



On the interval $[a,b]$ how many inflection points does the function $g(x)$ have? Explain

10. Evaluate the limit

$$\lim_{x\to 0}\left(\cos\left(x\right)\right)^{\frac{3}{x^2}}$$

a) e c) $\frac{-1}{2}$ e) $\frac{1}{\sqrt{e}}$ g) $\sqrt{e}$

b) $e^{-3/2}$ d) 1 f) 0 h) ∞

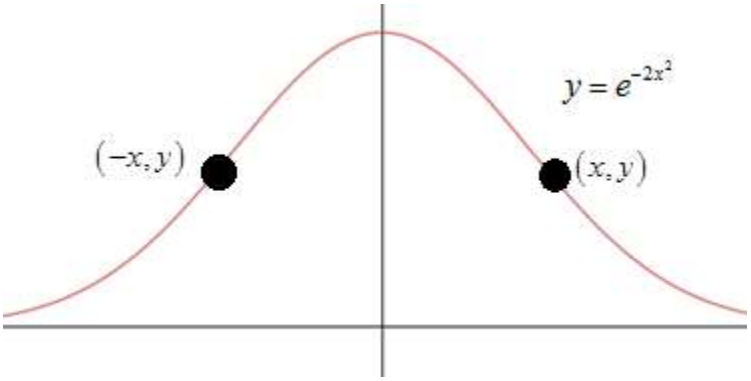
HINT: $\frac{\sin\left(\theta\right)}{\cos\left(\theta\right)}=\tan\left(\theta\right)$

This hint is not for the beginning of the problem.

11. A rectangle R has one side on the x-axis and two of its vertices on the graph of

$y = e^{-2x^2}$.

Find the largest possible area for R . In the interest of time, you can skip the work proving that you have an absolute extreme value.



12. Let

$$f'(x) = 3x^3 + x$$

a. Find the x-values of all local extrema

b. If $f(1) = 0$, find $f(\sqrt{2})$

13. Evaluate the definite integral

$$\int_4^9 \left(3\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$$

A) 10
B) 20
C) 30
D) 40
E) 60
F) 70
G) 80
H) None of these

14.

a. Evaluate the limit below.

$$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{6k}{n^2}+\sum_{k=1}^{n}\frac{30k^2}{n^3}\right)$$

A) 1
B) 2
C) 4
D) 8
E) 13
F) 15
G) 17
H) None of these

b. Let

$$g(x)=\int_{1}^{\sqrt{x}}\frac{4}{\sqrt{3t^2+1}}\,dt$$

Find $g'(1)$.

A) 1
B) 2
C) 3
D) 4
E) 6
F) 7
G) 0
H) None of these

15. Evaluate

$$\int_0^{\sqrt{\ln 2}} 8x\cosh\left(x^2\right)dx$$

A) 1
B) 2
C) 3
D) 4
E) 6
F) 7
G) 8
H) None of these

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