FINAL EXAM

Math 103
Dec 14, 2017

Name: ______________________

ID: ______________________

"My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this"

Signature: ______________________

Read all of the following information before starting the exam:

- Check your exam to make sure all 10 pages are present.
- You may use writing implements and a single handwritten 5"x8" notecard.
- DNE means "does not exist".
- NO CALCULATORS.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Good luck!

1	6		9	7	
2	6		10	7	
3	6		11	7	
4	6		12	7	
5	6		13	7	
6	7		14	7	
7	7		15	7	
8	7				
Total	100				

_____ **1.** Simplify as much as possible:

$$e^{\frac{1}{3}\ln 8} - 5\ln(\sqrt[4]{e}).$$

a. 0	e. 1/10
b. 1/2	f. 3/2
c. 1/5	g. 3/4
d. 1	h. 17/12

2. Solve the trigonometric equation for x. Find all all solutions $0 \le x \le 2\pi$.

$$2\sin x \cos x = \sqrt{3}\cos x.$$

_____ **3.** Find

$$\lim_{x \to -1} \frac{x^3}{(x+1)^2}.$$

a. 1	e. 0
b. -1	f. DNE because approaches ∞
c. $1/2$	g. DNE because approaches $-\infty$
d. $-1/2$	h. DNE because one-sided limits are different

4. Show that there is some x such that $\frac{1}{x} = \ln x$.

_____ **5.** Find the equation of the tangent line to the function below at $(1,1)$.

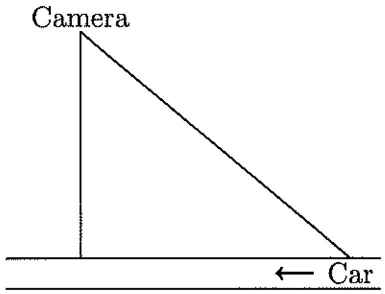
$$y^4 + xy = x^3 - x + 2.$$

a. $5x + 12y = 17$
b. $2x - 5y = -3$
c. $5x - 2y = 3$
d. $3x + 2y = 5$
e. $x - 5y = -4$
f. $x + 5y = 6$
g. $5x + y = 6$
h. $10x - 2y = 8$

6. Find

$$\frac{d}{dx} \ln \sin e^{\arctan x}.$$

7. A camera is located 20 feet from a straight road along which a car is traveling due west at 60 feet per second. The camera turns so that it is pointed at the car at all times. In radians per second, how fast is the camera turning when the car is 20 feet away from the point on the road closest to the camera.



______ **8.** Evaluate

$$\lim_{x \to 0} \frac{3\sin(x) - \sin(3x)}{x - \sin(x)}.$$

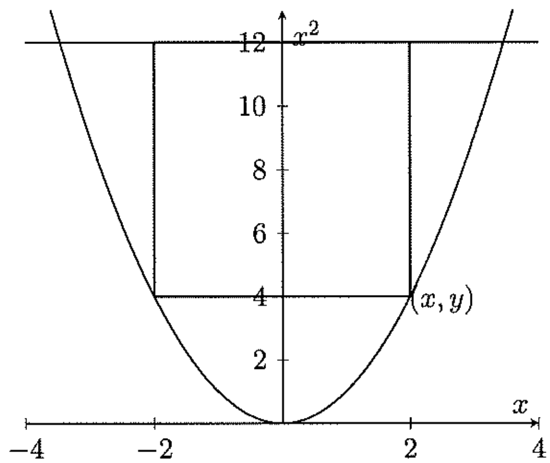
a. 24
b. 20
c. 16
d. 12
e. 10
f. 9
g. 8
h. 6

______ **9.** If $f(x) = a \ln x - a^3 x$ has a local minimum when $x = 4$ and $a \neq 0$, what is a?

a. $-1/4$
b. $1/4$
c. $-1/2$
d. $1/2$
e. -1
f. 1
g. -2
h. 2

_____ **10.** Find the area of the rectangle with maximum area that fits inside the parabola $y = x^2$ below the line $y = 12$ with the top side of the rectangle on the line $y = 12$. An example rectangle is drawn below.

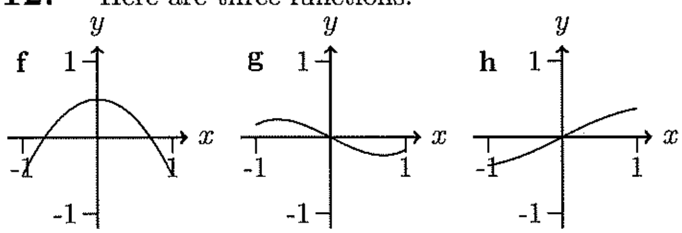
a. 44 e. 24
b. 36 f. 20
c. 32 g. 16
d. 28 h. 12



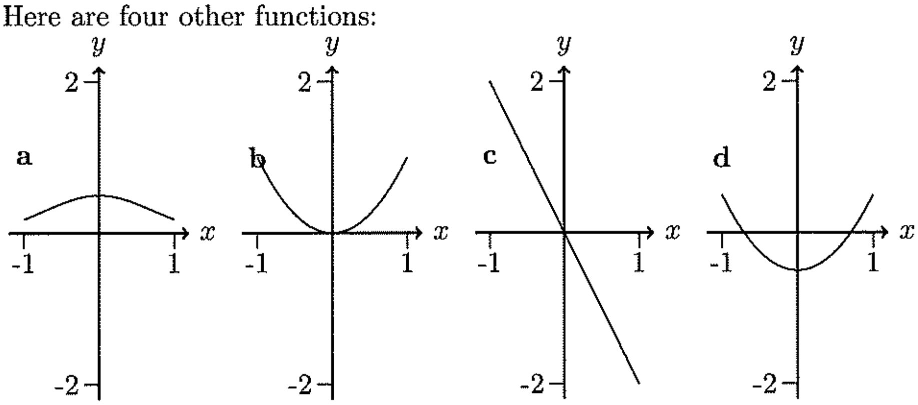
_____ **11.** What is the derivative of $x\sin(|x| + \pi/3)$ at 0?

a. 0	e. $\sqrt{3}/2$
b. 1	f. -1
c. $\sqrt{2}/2$	g. $-1/2$
d. $1/2$	h. Undefined

12. Here are three functions:



Here are four other functions:



- The derivative of f is:
- The derivative of g is:
- The derivative of h is:

_____ **13.** Approximate $\int_0^{12} \frac{24}{x} dx$ using 4 rectangles and the right endpoint method.

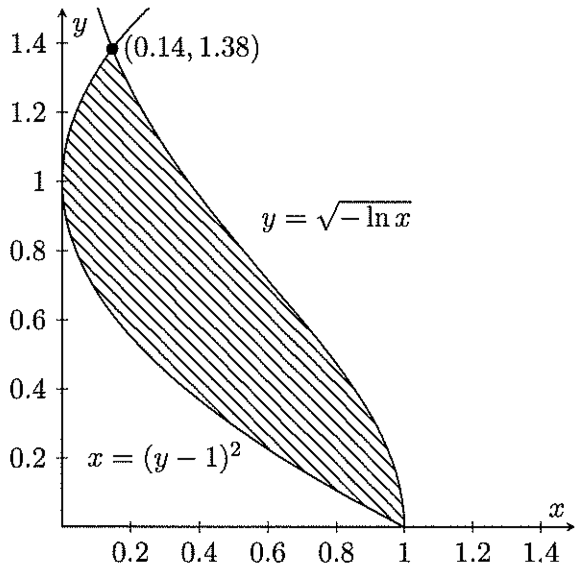
a. 20
b. 24
c. 25
d. 32
e. 36
f. 48
g. 50
h. 62

_____ **14.** Find

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx.$$

a. $\ln(x^{3/2}) + C$
b. $\ln\sqrt{x} + C$
c. $2\ln\sqrt{x} + C$
d. $\frac{1}{2}\ln\sqrt{x} + C$
e. $\ln(\sqrt{x}(1+\sqrt{x})) + C$
f. $2\ln(\sqrt{x}+1) + C$
g. $\ln(\sqrt{x}+1) + C$
h. $\frac{1}{2}\ln(\sqrt{x}+1) + C$

15. The shaded area below is the region between $y = \sqrt{-\ln x}$ and $x = (y-1)^2$.



Set up (but don't solve) an integral (or sum of integrals) which expresses the shaded area.