



MATH 104 FINAL EXAM FALL 2016

YOUR NAME:

YOUR TA'S NAME:

RECITATION NUMBER OR DAY/TIME:

SELECT YOUR SECTION / INSTRUCTOR:

MATH 104-001	MWF	10:00-11:00	RIMMER
MATH 104-002	TR	10:30-12:00	DETURCK
MATH 104-003	TR	1:30-3:00	STOVALL
MATH 104-004	MWF	1:00-2:00	PALVANNAN
MATH 104-005	TR	12:00-1:30	GRESSMAN

Please *turn off and put away all electronic devices*. You may use both sides of an 8.5" x 11" sheet of paper with handwritten notes. No other resources are allowed. **Show all work**. Please **clearly mark** a multiple choice option for each problem. Remember to fill out the identification information at the top of this page and sign on the line below.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

Your signature

FOR OFFICIAL USE (DO NOT WRITE BELOW THIS LINE)

QUESTION NUMBER	POINTS POSSIBLE	YOUR SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
TOTAL	150	

1. Find the length of the part of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ for $1 \leq x \leq \sqrt{2}$.

(a) $\frac{13}{16}$

(b) $\frac{11}{16}$

(c) $\frac{7}{8}$

(d) $\frac{13\sqrt{2}}{16}$

(e) $\frac{11\sqrt{2}}{16}$

(f) $\frac{7\sqrt{2}}{8}$

2. Find the y -coordinate of the centroid of the region in the upper half-plane (i.e., for $y > 0$) bounded by the semicircle $y = \sqrt{1 - x^2}$. (It is easiest to use a geometric formula to find the area of the region.)

(a) $\frac{4\pi}{3}$

(b) $\frac{4}{3\pi}$

(c) $\frac{7\pi}{3}$

(d) $\frac{7}{3\pi}$

(e) $\frac{28\pi}{9}$

(f) $\frac{28}{9\pi}$

3. Compute the value of the integral below.

$$\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$$

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

(f) none of these

4. Compute the constants A and B in the partial fractions expansion indicated below. To receive full credit, it is not necessary to compute C, D , or E .

$$\frac{x^4 + 16}{x^4 - 16} = A + \frac{B}{x - 2} + \frac{C}{x + 2} + \frac{Dx + E}{x^2 + 4}$$

(a) $A = -1, B = 1$

(b) $A = 0, B = 1$

(c) $A = 1, B = 1$

(d) $A = -1, B = -1$

(e) $A = 0, B = -1$

(f) $A = 1, B = -1$

5. Compute the indefinite integral indicated below. [Hint: Write $\frac{1}{\cos^2 \theta} = \sec^2 \theta$ and integrate by parts.]

$$\int \left(1 + \frac{\ln |\sin \theta|}{\cos^2 \theta} \right) d\theta$$

(a) $(\sin \theta) \ln |\sin \theta| + C$

(b) $(\cos \theta) \ln |\sin \theta| + C$

(c) $(\tan \theta) \ln |\sin \theta| + C$

(d) $(\csc \theta) \ln |\sin \theta| + C$

(e) $(\sec \theta) \ln |\sin \theta| + C$

(f) $(\cot \theta) \ln |\sin \theta| + C$

6. Only one of the following four improper integrals diverges. Choose that improper integral and justify why it diverges. (You need NOT justify why the other integrals converge.)

(a) $\int_2^{\infty} \frac{\arctan x}{1+x^3} dx$

(b) $\int_2^{\infty} \frac{1}{\sqrt{x^4+x^2}} dx$

(c) $\int_2^{\infty} \frac{1+\sin x}{x^2} dx$

(d) $\int_2^{\infty} \frac{1}{\sqrt[3]{x^2-1}} dx$

7. For a certain real number k , the function

$$f(X) = \begin{cases} \frac{k}{X^2 + 1} & \text{if } X \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for a continuous random variable X . For this value of k , find the probability that $X > 1$.

(a) 0

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) 1

(e) $\frac{1}{2}$

(f) $\frac{1}{4}$

8. Determine whether the sequence $a_n = (-1)^{n-1} \frac{n^2}{1+n^2+n^3}$ converges or diverges. If it converges, find its limit.

(a) divergent, $\lim_{n \rightarrow \infty} a_n = 0$

(b) convergent, $\lim_{n \rightarrow \infty} a_n = 1$

(c) convergent, $\lim_{n \rightarrow \infty} a_n = 0$

(d) convergent, $\lim_{n \rightarrow \infty} a_n = -1$

(e) divergent, $\lim_{n \rightarrow \infty} a_n = \infty$

(f) divergent, limit doesn't exist

9. Determine whether the following series are convergent or divergent. Justify your answers.

$$\text{I: } \sum_{n=1}^{\infty} \frac{n^2 - 3n}{\sqrt[3]{n^{10} - 4n^2}} \qquad \text{II: } \sum_{n=1}^{\infty} \frac{(-n)^n}{5^{2n+3}}$$

(a) I & II divergent

(b) I convergent, II divergent

(c) I divergent, II convergent

(d) I & II convergent

10. Determine whether the following series are convergent or divergent. Justify your answers.

$$\text{I: } \sum_{n=1}^{\infty} \frac{\arctan n}{n^4} \qquad \text{II: } \sum_{n=1}^{\infty} \frac{\sin \frac{1}{n}}{n^2}$$

(a) I & II divergent

(b) I convergent, II divergent

(c) I divergent, II convergent

(d) I & II convergent

11. Find the interval of convergence of the power series below.

$$\sum_{n=1}^{\infty} \frac{(4x-1)^n}{n^{\frac{3}{4}}(n^2+2)}$$

(a) $(0, \frac{1}{2}]$

(b) $[0, \frac{1}{2}]$

(c) $(0, \frac{1}{2})$

(d) $[0, \frac{1}{2})$

(e) $(-\frac{1}{2}, 0]$

(f) $(-\infty, \infty)$

12. Find the Taylor polynomial of degree 2 for $f(x) = \sqrt{x+16}$ centered at $x = 9$.

(a) $5 + \frac{4}{5}x + \frac{9}{250}x^2$

(b) $5 - \frac{3}{5}(x-5) + \frac{1}{125}(x-5)^2$

(c) $5 + \frac{1}{10}(x-9) - \frac{1}{1000}(x-9)^2$

(d) $5 + \frac{3}{5}(x-5) + \frac{8}{125}(x-5)^2$

(e) $5 + \frac{1}{5}(x-9) + \frac{16}{125}(x-9)^2$

(f) none of these

13. Let $y(x)$ be the solution of the initial value problem

$$x \frac{dy}{dx} = e^x - y \quad \text{with} \quad y(\ln 2) = 0.$$

Find $y(1)$.

(a) $\frac{e^2}{2}$

(b) $2e^2$

(c) $\frac{e}{2}$

(d) 0

(e) $e - 2$

(f) 1

14. Let $f(x)$ be a continuous function that satisfies $f(0) = 0$ and $f(x) > 0$ for $x > 0$. For every $b > 0$, when the region between the graph of $y = f(x)$, the x -axis, and the line $x = b$ is rotated around the x -axis, the volume of the resulting solid is $18\pi b^2$. What is $f(x)$?

(a) $9x$

(b) $3x^2$

(c) $6\sqrt{x}$

(d) $27x^{3/2}$

(e) $9x^2$

(f) $\sqrt{3x}$

15. A tank contains 100 gallons of water in which 300 pounds of salt is dissolved. At some initial time, workers begin pumping in fresh water, i.e., containing no salt, a rate of 10 gallons per minute. During this process, the tank is kept well-mixed and 20 gallons per minute of the resulting saltwater is pumped out of the tank (in particular, note that the tank will be empty after 10 minutes). Find the total amount of salt in the tank (measured in pounds) which remains 9 minutes after the process starts.

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

(f) 6