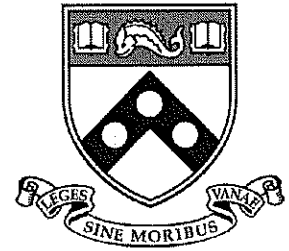


University of Pennsylvania

Math 104 Final Exam Fall 2015



First and Last Name _____ (PRINT) Penn ID _____

Professor (circle one): RIMMER DETURCK WONG WEBER HABLICSEK BLOCK

Recitation number _____

There are fifteen questions on this examination. No calculators are allowed, but you may use one standard sized 8.5"X11" sheet with notes handwritten on both sides. Show your work in the space provided, and then **transfer your answers carefully** to this sheet. A correct answer with little or no supporting work will receive little or no credit. Use the space provided to show all work. Two sheets of scrap paper are provided at the end of the exam, if you intend for the work on these pages or the work on the back of any page to be graded you must indicate this in some obvious way.

You have **2 hours** to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 2 hours have elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave.

Once you have completed the exam, sign the academic integrity statement below.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination. In particular, all the work on this test is my own.

Signature

- | | |
|---|-----------------------------|
| 1. (A) (B) (C) (D) (E) (F) | 8. (A) (B) (C) (D) (E) (F) |
| 2. (A) (B) (C) (D) (E) (F) | 9. (A) (B) (C) (D) (E) (F) |
| 3. (A) (B) (C) (D) (E) (F) | 10. (A) (B) (C) (D) (E) (F) |
| 4. (1) Conv. or Div. (2) Conv. or Div.
(3) Conv. or Div. (4) Conv. or Div. | 11. (A) (B) (C) (D) (E) (F) |
| 5. (A) (B) (C) (D) (E) (F) | 12. (A) (B) (C) (D) (E) (F) |
| 6. (A) (B) (C) (D) (E) (F) | 13. (A) (B) (C) (D) (E) (F) |
| 7. (A) (B) (C) (D) (E) (F) | 14. (A) (B) (C) (D) (E) (F) |
| | 15. (A) (B) (C) (D) (E) (F) |

1. The first few nonzero terms of the Maclaurin series for $f(x) = \ln(1 + \sin x)$ are

(a) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{24}x^3 + \dots$

(b) $1 + \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{48}x^3 + \dots$

(c) $x - \frac{1}{2}x^2 + \frac{1}{8}x^3 - \frac{1}{24}x^4 + \dots$

(d) $1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{6}x^4 + \dots$

(e) $x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 + \dots$

(f) $1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{12}x^4 + \dots$

2. If it converges, find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n} (2n)!}$. If the series diverges, explain why.

- (a) $\ln 2$ (b) $\ln 3 - \ln 2$ (c) $1/e^2$ (d) $1/2$ (e) $2/e$ (f) Diverges
-

3. For which values of x does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n4^n}$ converge?

(a) $-3 < x < 5$

(b) $-3 \leq x < 5$

(c) $-3 < x \leq 5$

(d) $-5 < x \leq 3$

(e) $-5 \leq x < 3$

(f) $-5 \leq x \leq 3$

4. Determine whether the following series converge or diverge?

$$(1) \sum_{n=1}^{\infty} \frac{n^3}{n^4 + 4} \quad (2) \sum_{n=1}^{\infty} \frac{3^n}{n!} \quad (3) \sum_{n=2}^{\infty} \frac{\ln(\ln(n))}{\ln n} \quad (4) \sum_{n=1}^{\infty} \frac{3n^2}{(n!)^2}$$

(1) converge or diverge (2) converge or diverge

(3) converge or diverge (4) converge or diverge

5. What is the limit of the sequence $\left\{n^2 \left(1 - \cos \frac{1}{n}\right)\right\}$?

(a) 1

(b) -1

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{1}{2}$

(e) $-\frac{\sqrt{3}}{2}$

(f) diverges

6. The solution of the initial-value problem: $x \frac{dy}{dx} + 3y = 7x^4$ $y(1) = 1$
satisfies $y(2) =$

(a) 0

(b) 1

(c) 2

(d) 4

(e) 8

(f) 16

7. The solution of the initial-value problem: $\frac{dy}{dx} - 20x^4e^{-y} = 0$ $y(0) = 0$
satisfies $y(1) =$

(a) $\ln 5$

(b) $\ln 4$

(c) $\ln 3$

(d) $\ln 2$

(e) 1

(f) 0

8. The function

$$f(x) = \begin{cases} \frac{k}{x^3} & 1 < x \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function for a certain value of k . For that probability density function, find the probability that $x > 2$

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{2}{3}$

(e) $\frac{1}{5}$

(f) $\frac{1}{6}$

9. An object moves in such a way that its acceleration at time t seconds is $\frac{1}{t^2 + 5t + 6}$ meters per second². If the initial velocity of the object is $\ln \frac{2}{3}$ meters per second, what is the limit of its velocity as $t \rightarrow \infty$?

- (a) $\ln \frac{3}{2}$ meters per second (b) $\ln 6$ meters per second (c) 1 meters per second
(d) $\ln \frac{4}{9}$ meters per second (e) $\ln \frac{9}{4}$ meters per second (f) 0 meters per second
-

10. $\int_0^{\pi/8} \tan^4 2x \sec^4 2x dx$

(a) $\frac{4}{9}$

(b) $\frac{7}{24}$

(c) $\frac{5}{14}$

(d) $\frac{9}{28}$

(e) $\frac{6}{35}$

(f) $\frac{1}{7}$

11. $\int_{\frac{1}{2}}^{\infty} \frac{\ln(2x)}{x^2} dx$

- (a) $1 - \ln 2$ (b) 2 (c) $\ln 2 - \frac{1}{2}$ (d) $\frac{1}{2}$ (e) $2 - 2 \ln 2$ (f) the integral diverges
-

12. Find the y -coordinate of the centroid of the region bounded by the x axis, the y axis, and the graph of $y = \cos x$ for $0 \leq x \leq \frac{\pi}{2}$, if the density is constant.

(a) $\frac{\pi}{18}$

(b) $\frac{\pi}{12}$

(c) $\frac{\pi}{8}$

(d) $\frac{\pi}{6}$

(e) $\frac{\pi}{4}$

(f) $\frac{\pi}{2}$

13. Find the length of the part of the curve $y = \frac{3}{16}e^{2x} + \frac{1}{3}e^{-2x}$ for $0 \leq x \leq \ln 2$.

(a) $\frac{13}{16}$

(b) $\frac{11}{16}$

(c) $\frac{3}{8}$

(d) $\frac{9}{8}$

(e) $\frac{29}{64}$

(f) $\frac{3}{4}$

14. The region between the graph of $y = 1 - x^2$ and the x -axis is rotated around the line $y = 1$. What is the volume of the resulting solid?

(a) $\frac{2\pi}{5}$

(b) $\frac{4\pi}{5}$

(c) $\frac{6\pi}{5}$

(d) $\frac{8\pi}{5}$

(e) 2π

(f) $\frac{12\pi}{5}$

15. Calculate the volume of the solid obtained by rotating the area between the graph of $y = \frac{1}{\sqrt{x^2 - 1}}$ and the x -axis for $1 < x < \sqrt{5}$ around the y -axis.

(a) π

(b) 4π

(c) 6π

(d) 8π

(e) 3π

(f) 2π

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