University of Pennsylvania
Math 104 Final Exam
Fall 2017

First and Last Name _____(PRINT) Penn ID_____

Professor (circle one): Ghini-Bettiol Sergel Block Gressman Rimmer

Recitation number _____

There are fifteen questions on this examination. No calculators are allowed, but you may use one standard sized 8.5"X11" sheet with notes handwritten on both sides. Show your work in the space provided, and then **transfer your answers** *carefully* to this sheet.
It is important to show your work because we will be going back over it – you might gain additional partial credit for substantial progress toward the solution of a problem, or you might *lose* credit for an unsubstantiated correct answer.
Please put away and silence (don't set to vibrate) all electronic devices (computers, tablets, cell phones, mp3 players), use of these are forbidden during the examination period. Good luck!

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination. In particular, all the work on this test is my own.

_____
Signature

1. (A) (B) (C) (D) (E) (F)

2. (A) (B) (C) (D) (E) (F)

3. (A) (B) (C) (D) (E) (F)

4. (A) (B) (C) (D) (E) (F)

5. (A) (B) (C) (D) (E) (F)

6. (A) (B) (C) (D) (E) (F)

7. (A) (B) (C) (D) (E) (F)

8. (A) (B) (C) (D) (E) (F)

9. (A) (B) (C) (D) (E) (F)

10. (A) (B) (C) (D) (E) (F)

11. (A) (B) (C) (D) (E) (F)

12. (A) (B) (C) (D) (E) (F)

13. (A) (B) (C) (D) (E) (F)

14. (A) (B) (C) (D) (E) (F)

15. (A) (B) (C) (D) (E) (F)

1. Find the volume of the solid generated by revolving the region bounded above by $y = \sin x$ and bounded below $y = 0$ for $0 \le x \le \pi$ about the line $x = \pi$.

(a) π^2 (b) $2\pi^2$ (c) $4\pi^2$ (d) $\frac{\pi^2}{2}$ (e) $\frac{\pi^2}{4}$ (f) None of these

2. Find the volume of the solid generated by revolving the region bounded above by $y = \sec x$ and bounded below $y = 0$ for $0 \le x \le \frac{\pi}{3}$ about the $x-$axis.

(a) π (b) 2π (c) $\sqrt{3}\,\pi$ (d) 3π (e) 4π (f) None of these

3. Let $y = \frac{x^4}{16} + \frac{1}{2x^2}$. Find the arclength for $1 \le x \le \sqrt{2}$.

(a) $\frac{5}{7}$ (b) $\frac{6}{7}$ (c) $\frac{5}{6}$ (d) $\frac{7}{4}$ (e) $\frac{7}{16}$ (f) None of these

4. Evaluate

$$\int_1^2 \frac{x^2+x+1}{x^2+x}\,dx.$$

(a) 0 (b) 1 (c) $1+\ln\left(\frac{4}{3}\right)$ (d) 2 (e) $2+\ln\left(\frac{8}{3}\right)$ (f) None of these

5. Evaluate

$$\int_1^2 x \ln\left(x^2+1\right) dx.$$

(a) 0 (b) 1 (c) $\ln(2)$ (d) $\frac{1}{2}$ (e) $\ln(2)-\frac{1}{2}$ (f) None of these

6. Evaluate

$$\int_0^3 \frac{dx}{\left(25-x^2\right)^{3/2}} \ .$$

(a) 0 (b) $\frac{1}{100}$ (c) $\frac{3}{100}$ (d) $\frac{5}{100}$ (e) $\frac{7}{100}$ (f) None of these

7. Let $y(x)$ be the solution to the initial value problem

$$x\frac{dy}{dx} = y + x^2 \sin(x) \quad \text{with } y(\pi) = 0$$

What is $y(2\pi)$?

(a) $-\pi$ (b) -2π (c) -4π (d) 0 (e) 2π (f) 4π

8. Consider the initial value problem

$$\left(1+x^2\right)\frac{dy}{dx}=2y \quad \text{with } y(0)=2.$$

What is the $\lim_{x\to\infty} y(x)$?

(a) $2e^{\pi}$ (b) $2e^{\pi/2}$ (c) $2e^{\pi/4}$ (d) 1 (e) 0 (f) e^{π}

9. Let

$$f(r)=\begin{cases} Cr^2e^{-2r/b} & r\geq 0 \\ 0 & r<0 \end{cases}$$

Find C so that is a probability density function (pdf)
for the random variable r, $(b$ is a constant$)$.
This is used to model the distance between the nucleus and the electron
in a hydrogen atom. With $b>0,$ it is called the Bohr length.
Find the mean of this pdf.

(a) $C=\dfrac{b^3}{4},\ \text{mean}=b$ (b) $C=\dfrac{4}{b^2},\ \text{mean}=b$ (c) $C=\dfrac{4}{b},\ \text{mean}=b^2$

(d) $C=\dfrac{4}{b^3},\ \text{mean}=\dfrac{3}{2}b$ (e) $C=\dfrac{4}{b^2},\ \text{mean}=\dfrac{3}{2}b^2$ (f) $C=\dfrac{4}{b},\ \text{mean}=\dfrac{3}{2}b^3$

10. Find the limit of the sequence

$$a_n = \left\{ n\left[\ln\left(n+3\right) - \ln\left(n\right) \right] \right\}$$

(a) 0 (b) 1 (c) $\ln(3)$ (d) 3 (e) ∞ (f) the limit does not exist

11. Determine which of the following series are convergent.

For full credit be sure to explain your reasoning and tell what test was used.

$$(I)\sum_{n=2}^{\infty} 2ne^{-n^2} \qquad (II)\sum_{n=2}^{\infty} \frac{n+2\ln(n)}{2n^4} \qquad (III)\sum_{n=2}^{\infty} \frac{n^n}{n!}$$

(a) only (I) (b) only (I) and (II) (c) only (I) and (III)

(d) only (II) (e) only (II) and (III) (f) only (III)

12. Determine whether the following series convergent absolutely A, converge conditionally C, or diverge D. For full credit be sure to explain your reasoning and tell what test was used.

$$\sum_{n=2}^{\infty} \frac{(-1)^n 2^{2n}}{3^n} \qquad \sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

(a) both A (b) one A, the other C (c) one A, the other D
(d) both C (e) one C, the other D (f) both D

13. Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{2^n (x+5)^n}{\sqrt[3]{n}}.$$

(a) $\left[\frac{-11}{2}, \frac{-9}{2}\right]$ (b) $\left[\frac{-11}{2}, \frac{-9}{2}\right)$ (c) $\left(\frac{-11}{2}, \frac{-9}{2}\right)$ (d) $\left[\frac{9}{2}, \frac{11}{2}\right)$ (e) $\left(\frac{9}{2}, \frac{11}{2}\right)$ (f) $\left[\frac{9}{2}, \frac{11}{2}\right]$

14. Use the Taylor polynomial of degree 3 for $f(x) = \ln(1+x)$ centered at $x_0 = 0$ to approximate the value of $\ln\left(\frac{3}{2}\right)$.

$\ln\left(\frac{3}{2}\right) \approx$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{15}{4}$ (d) $\frac{5}{12}$ (e) $\frac{9}{24}$ (f) $\frac{11}{24}$

15. Let $F(x)$ be the unique function that satisfies $F(0)=0$ and $F'(x)=\frac{1}{x}\sin\left(x^3\right)$ for all x. Find the Taylor Series of $F(x)$ centered at $x_0=0$.

(a) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n+3}}{(2n+1)!}$

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n (6n+3)x^{6n+2}}{(2n+1)!}$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n+3}}{(6n+3)(2n+1)!}$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n+2}}{(2n+1)!}$

(e) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n (6n+2)x^{6n+2}}{(2n+1)!}$

(f) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+3}}{(6n+3)(2n+1)!}$

Scrap Paper

If you use this page and intend for it to be graded, then you must indicate so on the page with the original problem on it. On this page, make sure you label your work with the corresponding problem number.

Do **NOT** rip this page off.

Scrap Paper

If you use this page and intend for it to be graded, then you must indicate so on the page with the original problem on it. On this page, make sure you label your work with the corresponding problem number.

Do **NOT** rip this page off.