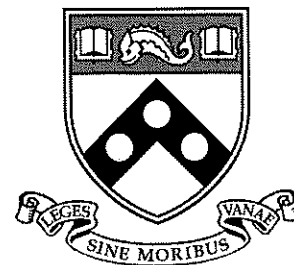


# University of Pennsylvania

## Math 104 Final Exam Spring 2016



First and Last Name \_\_\_\_\_ (PRINT) Penn ID \_\_\_\_\_

Professor (circle one):    **Xiao**                      **Rimmer**                      **Pimsner**

This exam has 15 multiple choice questions. Each question is worth 10 points for a total of 150 points. Partial credit will be given for the entire exam so be sure to show all work. Circle the correct answer and give supporting work, a correct answer with little or no supporting work will receive little or no credit. Use the space provided to show all work. A sheet of scrap paper is provided at the end of the exam. If you write on the back of any page please indicate this in some way.

You have **120 minutes** to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave.

Do **NOT** write in the grid below. It is for grading purposes only.

Problem	Points	Problem	Points
1		9	
2		10	
3		11	
4		12	
5		13	
6		14	
7		15	
8			
Total			

**My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.**

\_\_\_\_\_

**Name (printed)**

\_\_\_\_\_

**Signature**

1. Find the volume of the solid generated by revolving the region bounded by  $y = e^x$ ,  $y = 0$ ,  $x = 0$  and  $x = \ln 3$  about the  $x$ -axis.

(A)  $\frac{\pi}{3}$

(C)  $2\pi$

(E)  $\frac{\pi}{4}$

(G)  $4\pi$

(B)  $\frac{\pi}{2}$

(D)  $\frac{\pi}{6}$

(F)  $6\pi$

(H) None of these

2. Find the volume of the solid generated by revolving the region bounded by  $y = \sqrt{x}$ ,  $y = 0$ ,  $x = 1$  and  $x = 4$  about the  $y$ -axis.

(A)  $\frac{324\pi}{3}$

(C)  $\frac{62\pi}{5}$

(E)  $\frac{844\pi}{5}$

(G)  $\frac{211\pi}{5}$

(B)  $\frac{125\pi}{4}$

(D)  $\frac{31\pi}{5}$

(F)  $\frac{124\pi}{5}$

(H) None of these

3. Find the  $x$  coordinate of the center of mass of the lamina with constant density  $\delta$  bounded by the graphs of  $y = 2 - x^2$  and  $y = x$  for  $0 \leq x \leq 1$ .

(A)  $\frac{7}{6}$

(C)  $\frac{7}{3}$

(E)  $\frac{12}{5}$

(G)  $\frac{14}{3}$

(B)  $\frac{38}{45}$

(D)  $\frac{5}{14}$

(F)  $\frac{5}{12}$

(H) None of these

4. Evaluate

$$\int_{-\infty}^3 (x+3)e^x dx$$

- (A)  $e^3$       (C)  $2e^3$       (E)  $3e^3$       (G)  $4e^3$   
(B)  $5e^3$       (D)  $6e^3$       (F)  $\infty$       (H) None of these

5. Evaluate

$$\int_0^{\frac{\pi}{4}} 70 \sin^3(2x) \cos^4(2x) dx$$

- (A) 1      (C) 2      (E) 3      (G) 4  
(B) 5      (D) 6      (F) 7      (H) None of these

6. Compute

$$\int \frac{1}{x^2 \sqrt{x^2 + 4}} dx$$

- (A)  $-\frac{\sqrt{x^2 + 4}}{4x} + C$     (C)  $x\sqrt{x^2 + 4} + C$     (E)  $\frac{\sqrt{x^2 + 4}}{8} + C$     (G)  $4x\sqrt{x^2 + 4} + C$   
(B)  $-2x\sqrt{x^2 + 4} + C$     (D)  $\frac{\sqrt{x^2 + 4}}{2x} + C$     (F)  $-\frac{x}{4\sqrt{x^2 + 4}} + C$     (H) None of these

7. Evaluate

$$\int_4^6 \left( x - \frac{6-4x}{(x-3)(x^2-2x)} \right) dx.$$

- (A)  $\ln\left(\frac{1}{3}\right)$       (C)  $\ln\left(\frac{2}{3}\right)$       (E)  $10 - \ln(6)$       (G)  $10 - \ln 4$   
(B)  $8 - \ln 4$       (D)  $2 - \ln 2$       (F)  $10 - \ln(3)$       (H) None of these



8. Let

$$f(x) = \begin{cases} ax^2 & \text{if } -1 \leq x \leq 0 \\ bx & \text{if } 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

For which value of  $(a,b)$  is  $f(x)$  a probably density function

I.  $(a,b) = (3,0)$     II.  $(a,b) = (-3,4)$     III.  $(a,b) = (2,1)$     IV.  $(a,b) = \left(\frac{3}{2}, 1\right)$

(A) I only    (C) II only    (E) III only    (G) IV only

(B) I and IV only    (D) I, II, and IV only    (F) II and III only    (H) None of these

9. Solve the initial value problem

$$x \frac{dy}{dx} = e^x - y \quad \text{with } y(\ln 2) = 0$$

Find  $y(1)$ .

- (A)  $\frac{e^2}{2}$       (C)  $\frac{e}{2}$       (E)  $e-2$       (G)  $e^2$   
(B)  $2e^2$       (D) 0      (F) 1      (H) None of these

10. Solve the initial value problem

$$\frac{dy}{dx} = \frac{3x}{1+2y} \quad \text{with } y(-2) = 0$$

When  $y = 2$ , find the positive value of  $x$ .

(A)  $\frac{\sqrt{2}}{2}$       (C)  $\sqrt{2}$       (E)  $2\sqrt{2}$       (G) 1

(B)  $\sqrt{3}$       (D)  $-2$       (F) 2      (H) 0

11. Find the limit of the sequence

$$a_n = \left\{ \left( \frac{n}{n-2} \right)^{\frac{3n}{2}} \right\}$$

- (A)  $e^{1/3}$       (C)  $e^3$       (E)  $e$       (G)  $e^{-1}$   
(B)  $e^{-3}$       (D) 1      (F)  $e^2$       (H)  $\infty$

12. Determine if the series below converge absolutely  $\mathcal{A}$ , converge conditionally  $\mathcal{C}$ , or diverge  $\mathcal{D}$ .

I.  $\sum_{n=2}^{\infty} \frac{(-1)^n (n-3)^n}{(n)^n}$

II.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{e^n}{(\sqrt{3})^n}$

III.  $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\ln n}$

(A) I.  $\mathcal{A}$  II.  $\mathcal{C}$  III.  $\mathcal{D}$

(B) I.  $\mathcal{A}$  II.  $\mathcal{D}$  III.  $\mathcal{C}$

(C) I.  $\mathcal{C}$  II.  $\mathcal{A}$  III.  $\mathcal{D}$

(D) I.  $\mathcal{C}$  II.  $\mathcal{D}$  III.  $\mathcal{A}$

(E) I.  $\mathcal{D}$  II.  $\mathcal{C}$  III.  $\mathcal{A}$

(F) I.  $\mathcal{D}$  II.  $\mathcal{C}$  III.  $\mathcal{A}$

(G) I.  $\mathcal{D}$  II.  $\mathcal{D}$  III.  $\mathcal{C}$

(H) I.  $\mathcal{A}$  II.  $\mathcal{D}$  III.  $\mathcal{D}$

13. The interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} (x+1)^n$$

- (A)  $(-2, 0]$       (E)  $(-4, 0]$   
(B)  $[-2, 0]$       (F)  $[0, 2]$   
(C)  $(-2, 0)$       (G)  $\{-1\}$   
(D)  $[-2, 0)$       (H)  $(-\infty, \infty)$

14. Find the Taylor polynomial  $P_2(x)$  of order two for  $f(x) = e^{\cos x}$  centered at  $x = \frac{\pi}{2}$  and evaluate it at  $x = 0$ .  $P_2(0) =$

(A)  $1 + \frac{\pi}{4} + \frac{3\pi^2}{8}$

(E)  $1 + \pi + \frac{\pi^2}{2}$

(B)  $1 + \frac{\pi}{2} + \frac{3\pi^2}{16}$

(F)  $1 - \frac{\pi}{2} + \frac{\pi^2}{4}$

(C)  $1 + \frac{\pi}{2} + \frac{\pi^2}{8}$

(G)  $1 - \frac{\pi}{2} + \frac{\pi^2}{16}$

(D)  $1 + \frac{\pi}{4} + \frac{\pi^2}{16}$

(H) None of these

15. Find the first three nonzero (and non-constant) terms in the Maclaurin series for

$$\int \frac{\sin(x^2)}{e^x} dx$$

(A)  $C + \frac{x^2}{2} - \frac{7x^4}{24} + \frac{5x^6}{36}$

(B)  $C + \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4}$

(C)  $C + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{10}$

(D)  $C - \frac{x^3}{3} - \frac{x^5}{5} - \frac{x^7}{7}$

(E)  $C + \frac{x^3}{2} - \frac{x^4}{3} + \frac{x^5}{4}$

(F)  $C - \frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5}$

(G)  $C + \frac{x^3}{6} - \frac{x^4}{24} + \frac{x^5}{120}$

(H) None of these



**Scrap Paper**

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