Final Exam - Math 114 - Fall 2017

Each problem is worth 10 points. Circle your answers. Show your work - correct answers with little or no supporting work will receive little or no credit. Partial credit may be given for wrong answers if there is significant progress towards a solution.

Name: (please print) ______________________________

Circle the name of your lecturer: Donagi Haglund Hilburn Jang

TA's Name: (please print) ______________________________

Day of week and time of recitation:______________________

My signature below certifies that I have complied with Penn's Code of Academic Integrity in completing this exam

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Signature

The table below is for grading purposes - do not write below this line

1.__________ 8.________

2.__________ 9.________

3.__________ 10.________

4.__________ 11.________

5.__________ 12.________

6.__________ 13.________

7.__________

Total __________

1. Compute the volume of the parallelepiped determined by the three vectors

$$\vec{u} = \langle -2, 3, 4 \rangle \qquad \vec{v} = \langle 0, 2, 1 \rangle \qquad \vec{w} = \langle 5, 5, 3 \rangle .$$

(A) -23 (B) -27 (C) -57

(D) 23 (E) 27 (F) 53

(G) 57

2. Find the values a and b that make $\vec{r}(t)$ continuous everywhere.

$$\vec{r}(t) = \begin{cases} t\sin\left(\frac{1}{t^2}\right)\vec{\imath} + a\vec{\jmath} & \text{, for } t < 0 \\ (2t+b)\vec{\imath} + e^t\vec{\jmath} & \text{, for } t \geq 0. \end{cases}$$

(A) $a = 1,\ b = 0$ **(B)** $a = 1,\ b = 1$ **(C)** $a = -1,\ b = 2$

(D) $a = 1,\ b = -1$ **(E)** $a = \infty,\ b = 0$ **(F)** $a = 2,\ b = 1$

(G) such a and b do not exist

3. Let $\vec{r}(t)$ be the position of a particle starting at $t = 0$.

$$\vec{r}(t) = \left\langle 2\cos t, 2\sin t, \sqrt{5}t \right\rangle$$

Find the time t when the distance traveled by the particle is 5 units.

(A) $2/3$	**(B)** 1	**(C)** $3/5$
(D) $5/7$	**(E)** 2	**(F)** $\sqrt{5}/2$
(G) $2\sqrt{5}/3$	**(H)** $5/3$	

4. Consider the function $f(x, y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y$. Circle any points below which are saddle points of f:

(A) $(3, 1)$ **(B)** $(0, 1)$ **(C)** $(3, -2)$ **(D)** $(-2, 3)$

(E) $(0, -2)$ **(F)** $(1, 0)$ **(G)** $(1, 3)$

5. Find the absolute maximum and the absolute minimum of $f(x, y) = x^2 + xy - y^2$ over the region $0 \leq x \leq 4$, $0 \leq y \leq 3$. The sum of the absolute maximum and the absolute minimum is

(A) 10 (B) 11 (C) 12 (D) 13

(E) 16 (F) 19 (G) 20

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6. Consider the sphere S cut out by $x^2 + y^2 + z^2 = 2$. Maximize $(D_{\vec{u}}f)|_P$ where $f(x, y, z) = -x + 2y + 3z$ and $\vec{u}$ is a unit vector in the tangent plane to S at the point $P = (1, 0, 1)$.

(A) $\sqrt{3}$ **(B)** $1 + 2\sqrt{2}$ **(C)** $2 + \sqrt{3}$ **(D)** $\sqrt{2}$

(E) $2\sqrt{3}$ **(F)** $2 + \sqrt{2}$ **(G)** $3\sqrt{2}$

7. Find the volume of the wedge-shaped region bounded above by the plane $z = x$ and below by the xy-plane (i.e., satisfying $0 \leq z \leq x$) and contained in the cylinder $x^2 + y^2 = 9$.

(A) 18 (B) 5π (C) $12 + e$

(D) 20 (E) $3\sqrt{21}$ (F) 7π

(G) $3\sqrt{20}$

8. Find the z-coordinate of the center of mass of the first octant of the unit sphere (that is, the region $x^2 + y^2 + z^2 \leq 1$, $x \geq 0, y \geq 0, z \geq 0$) with mass density $\delta(x, y, z) = y$.

(A) $3/(2\pi)$ **(B)** $1/2$ **(C)** $\pi/(9 \ln 2)$

(D) $1 - (1/\pi)$ **(E)** $16/(15\pi)$ **(F)** $1/e$

(G) $2/3$

9. Use the change of variables $v = 4y - x$, $u = x - y$ to evaluate

$$\int\int_R e^{4y-x}\,dx\,dy,$$

where R is the parallelogram in the xy-plane with vertices $(0,0), (3,3), (4,1)$ and $(7,4)$.

(A) $e^{15} - 2$ (B) $(e^{15} - 1)/15$ (C) $e^9 - 1$

(D) $8\pi^2$ (E) $17\sqrt{e^{13} - 4}$ (F) 101

(G) 0

10. Calculate

$$\oint_C \mathbf{F} \cdot d\mathbf{r}$$

(the circulation of the field $\mathbf{F}$ around the curve C in the indicated direction), where

$$\mathbf{F} = \langle e^{x^2} + z, x + \cos(3y) + z, 2y + 4z \rangle$$

and C is the closed contour consisting of three straight line segments from $(3, 0, 0)$ to $(0, 4, 0)$ to $(0, 0, 6)$ and back to $(3, 0, 0)$.

(A) 0	**(B)** 6	**(C)** 12	**(D)** 18
(E) 27	**(F)** 31	**(G)** 54	

11. Find the outward flux of $\mathbf{F}$ across the boundary of the region D, where

$$\mathbf{F} = \langle x^3, \frac{6y^5}{5} + 3y^3z^2, \frac{6z^5}{5} + y^2z^3 \rangle$$

and D is the region cut from the solid cylinder $y^2 + z^2 \leq 1$ by the planes $x = -1$ and $x = 1$.

(A) 0	(B) π	(C) 2π	(D) 3π
(E) 4π	(F) 6π	(G) 8π	(H) 12π

12. Let C be the closed contour consisting of the six straight line segments from $(3, 0)$ to $(0, 4)$ to $(-2, 5)$ to $(1, 1)$ to $(-4, -1)$ to $(3, -3)$ and back to $(3, 0)$. You want to estimate

$$\oint_C \frac{x}{x^2 + y^2} dy - \frac{y}{x^2 + y^2} dx.$$

In which of the following intervals does it fall?

(A) $(-20, -10]$

(B) $(-10, -5]$

(C) $(-5, -1]$

(D) $(-1, 1)$

(E) $[1, 5)$

(F) $[5, 10)$

(G) $[10, 20)$

(H) none of the above

13. Find the work done by the force $\mathbf{F} = (x + 3y, y + z, 0)$ when travelling along the path C obtained as the intersection of the surfaces $y = x^2$ and $z = x + y$ from the point $(0, 0, 0)$ to $(1, 1, 2)$.

(A) $2\frac{1}{2}$	**(B)** $3\frac{1}{6}$	**(C)** $2\frac{2}{3}$	**(D)** $2\frac{1}{3}$
(E) $1\frac{2}{3}$	**(F)** $3\frac{2}{3}$	**(G)** $3\frac{1}{3}$	**(H)** $4\frac{1}{6}$