Math 114 - Final Exam with Solutions - University of Pennsylvania

- **Name:** _____
- **Your Professor (circle one) :** Mi Young Jang Florian Pop Robert Strain Henry Towsner Chenglong Yu

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this exam. I certify that all of the work on this test is my own.

Signature: _____

INSTRUCTIONS:

1. Check your exam to make sure all 17 pages are present.
2. You may write on the back of pages, and also use the blank pages at the end of the exam for additional space. Indicate clearly where your work can be found.
3. Complete the information requested above.
4. You may use writing implements and a single handwritten sheet of 8.5"x11" paper.
5. *No calculators, electronic devices, books or other aids are allowed.*
6. Show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit. We reserve the right to take off points if we cannot see how you arrived at your answer (even if your final answer is correct).
7. Good luck!

OFFICIAL USE ONLY:

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Points	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Your score														

Your total score: _____

1. Let $\mathbf{v}$ and $\mathbf{w}$ be vectors with $|\mathbf{v}| = 3$ and $|\mathbf{w}| = \sqrt{2}$. If the angle between $\mathbf{v}$ and $\mathbf{w}$ is $\pi/4$, what is $|\mathbf{v} + 2\mathbf{w}|^2$?

(a) 0 (b) 23 (c) 29 (d) 33
(e) 36 (f) 41 (g) 49 (h) None of the above

Answer: _____

2. Where does the x-axis intersect the line tangent to the curve $\mathbf{r}(t) = \langle e^{t-1}, t^2-t+2\rangle$ when $t = 1$?

(a) -2 (b) -1 (c) 0 (d) 1
(e) 2 (f) 3 (g) 4 (h) None of the above

Answer: _____

3. Find the arc length of the parameterized path $\mathbf{r}(t) = \langle \frac{3t}{2}, 2t^{3/2}, \frac{3\sqrt{3}t}{2} \rangle$ for $0 \leq t \leq 3$.

(a) 0 (b) 2 (c) 3 (d) 10
(e) 14 (f) 22 (g) 42 (h) None of the above

Answer: _____

4. Find the constant k that makes the function

$$f(x,y) = \begin{cases} \cos\left(\dfrac{xy^5 - x^5y}{x^2+y^2}\right), & (x,y) \neq (0,0) \\ \\ k\,, & (x,y) = (0,0) \end{cases}$$

continuous at $(0,0)$.

(a) -2 (b) $\frac{2}{3}$ (c) 1 (d) $-\frac{1}{2}$
(e) 0 (f) $-\frac{3}{2}$ (g) No such value of k exists (h) None of the above

Answer: _____

5. Consider the function $f(x, y) = x^3 + y^3 + x^2 + 3y^2 - 5x + 2018$. Find all local extrema and saddle points. Then f has

(a) 2 local maxima, 1 local minima, and 1 saddle points.
(b) 1 local maxima, 2 local minima, and 1 saddle points.
(c) 0 local maxima, 2 local minima, and 2 saddle points.
(d) 1 local maxima, 1 local minima, and 2 saddle points.
(e) 0 local maxima, 0 local minima, and 4 saddle points.
(f) 2 local maxima, 2 local minima, and 0 saddle points.
(g) 2 local maxima, 0 local minima, and 2 saddle points.
(h) None of the above

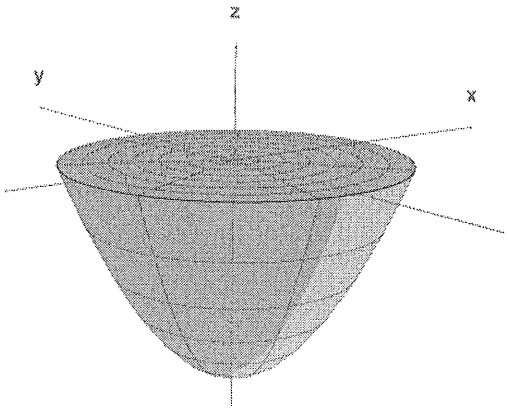
Answer: _____

6. Find the sum of the minimum and the maximum of $x+y+z$ on the ellipsoid $(x-1)^2+y^2+z^2 = 1$.

(a) 1 (b) 2 (c) 3 (d) $\sqrt{2}$
(e) $2\sqrt{2}$ (f) $\sqrt{3}$ (g) $2\sqrt{3}$ (h) None of the above

Answer: _____

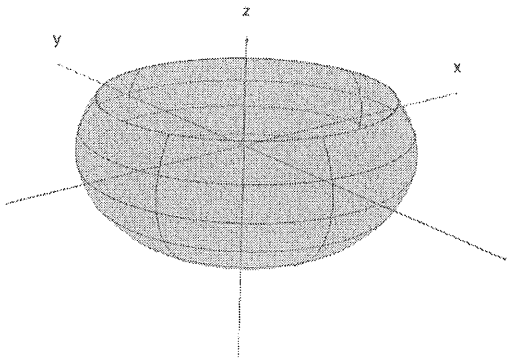
7. Find the total mass of the region bounded below by $z = x^2 + y^2 - 4$ and above by the xy-plane with density $f(x, y, z) = x^2 + y^2$



(a) $\frac{32\pi}{3}$
(b) $\frac{16\pi}{3}$
(c) $\frac{8\pi}{3}$
(d) $\frac{16}{3}$
(e) $\frac{64}{15}$
(f) $\frac{32}{15\pi}$
(g) $\frac{24\pi}{3}$
(h) None of the above

Answer: _____

8. Find the volume of the part of the ball $x^2 + y^2 + z^2 \leq 4$ below the plane $z = 1$.



(a) 9π (b) $\frac{5\pi}{3}$ (c) $\frac{32\pi}{3}$ (d) $\frac{16\pi}{3}$
(e) 4π (f) 2π (g) 8π (h) None of the Above

Answer: _____

9. Find the iterated integral

$$\int_0^1 \int_{\sqrt{y}}^1 ye^{x^5}\, dx\, dy$$

(a) $\frac{1}{10}$ (b) $\frac{1}{10}(e-1)$ (c) $e-1$ (d) 1
(e) $\frac{1}{4}$ (f) $\frac{1}{4}(e-1)$ (g) $\frac{1}{4}e$ (h) None of the Above

Answer: _____

10. Compute the circulation of $\vec{V} = \left(\sin(e^{\sqrt{1+x}}) - x^2y\right)\vec{\imath} + \left(\sin^5(\sqrt{1-y}) + xy^2\right)\vec{\jmath}$ along the unit circle centered at the origin, traveled anti-clockwise.

(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) $2\pi/3$
(e) π (f) $4\pi/3$ (g) $3\pi/2$ (h) None of the Above

Answer: _____

11. Evaluate the work (equivalently, the flow) of the vector field $\vec{V} = \ln(yz)\vec{\imath} + (x/y)\vec{\jmath} + (x/z)\vec{\kappa}$ along the curve C defined by $\vec{r}(t) = (t^2, e^t, e^{2t})$, where $1 \leqslant t \leqslant 3$.

(a) -1 (b) 0 (c) 1 (d) 24
(e) 50 (f) 54 (g) 78 (h) None of the Above

Answer: _____

12. Evaluate the surface integral

$$\iint_S (x^2 z + y^2 z)\, d\sigma$$

where S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$.

(a) 11π (b) 12π (c) 13π (d) 14π
(e) 15π (f) 16π (g) 17π (h) None of the Above

Answer: _____

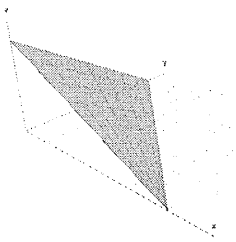
13. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where

$$\mathbf{F}(x, y, z) = (e^{x^2} + y^2)\mathbf{i} + (y + z^2)\mathbf{j} + (\sin(z^2) + x^2)\mathbf{k}$$

and C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$. Here C is oriented counterclockwise when viewed from above.



(a) 2 (b) 1 (c) 0 (d) -1
(e) -2 (f) -3 (g) -4 (h) None of the Above

Answer: _____

14. Use the divergence theorem to calculate the flux of $\mathbf{F}$ out across the surface S where

$$\mathbf{F}(x, y, z) = (3x^3 + 3xy^2)\mathbf{i} + (2y^3 + 8e^y \sin z)\mathbf{j} + (3z^3 + 8e^y \cos z)\mathbf{k}$$

and S is the surface of the solid between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 2$.

(a) $\frac{36}{5}\pi\left(4\sqrt{2} - 1\right)$ (b) $\frac{36}{5}\pi\left(4\sqrt{2} - 2\right)$ (c) $\frac{72}{5}\pi\left(4\sqrt{2} - 1\right)$ (d) $\frac{72}{5}\pi\left(4\sqrt{2} - 2\right)$
(e) $\frac{88}{5}\pi\left(4\sqrt{2} - 1\right)$ (f) $\frac{88}{5}\pi\left(4\sqrt{2} - 2\right)$ (g) $\frac{88}{5}\pi$ (h) None of the Above

Answer: _____

Extra space for work 1:

Extra space for work 2: