FINAL EXAM

Math 114
May 8, 2018

Name: _____

ID: _____

"My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this"

Signature: _____

Professor: (circle one)
Rimmer Simmons Towsner

Read all of the following information before starting the exam:

- Check your exam to make sure all 12 pages are present.
- You may use writing implements and a single handwritten sheet of 8.5"x11" paper.
- NO CALCULATORS.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Good luck!

Question	Max Points	Points	Question	Max Points	Points
1	15		8	15	
2	15		9	15	
3	15		10	15	
4	15		11	15	
5	15		12	15	
6	15		13	15	
7	15				
Total	195				

____ **1.** If $\mathbf{r}(t) = \langle \cos t, 2, \sin t \rangle$, what is the difference between the curvature and torsion, $\kappa - \tau$?

a. 0
b. 1/3
c. 1
d. $\pi/2$
e. 3
f. π
g. 2π
h. None of the above

____ **2.** What is the cosine of the angle between the vectors $\langle 1, 2, 2 \rangle$ and $\langle 2, 2, -1 \rangle$?

a. 0
b. $\frac{1}{3}$
c. $\frac{4}{9}$
d. $\frac{1}{2}$
e. $\frac{5}{9}$
f. $\frac{2}{3}$
g. $\frac{14}{27}$
h. None of the above

____ **3.** What is the x-intercept of the line tangent to the curve $\mathbf{r}(t) = \langle e^{t-1}, t^2 - t + 1\rangle$ when $t = 1$?

a. -2
b. -1
c. 0
d. 1
e. 2
f. 3
g. 4
h. None of the above

____ **4.** A shell is fired from the ground at an angle of $\pi/4$ with an initial velocity of 32ft/sec. The wind is blowing horizontally in the opposite direction, adding an acceleration of -2ft/sec. How far away does the shell land? (Assume the gravitational force is 32 ft/sec/sec.)

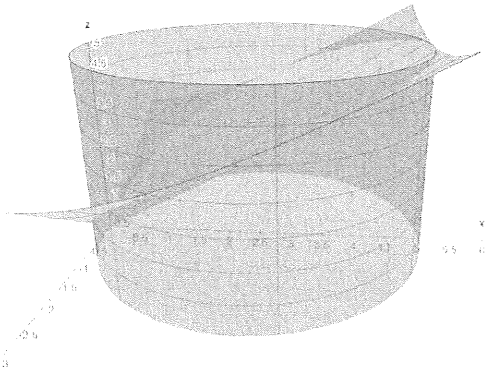
a. 0ft
b. 15ft
c. 16ft
d. 30ft
e. 32ft
f. 34ft
g. 38ft
h. None of the above

5. A flat plate has the shape of the ellipse $4x^2 + y^2 \leq 4$. The plate, including the boundary, is heated so that the temperature at any point (x, y) is $T(x, y) = x^2 + 2y^2 - 2x$. Locate the x and y-coordinates of the hottest and coldest points on the plate (you don't need to simplify the numbers).

6. A thin object is shaped like a circle with radius 5 meters. The density is proportional to distance from the outer edge, so the density is 0 at the outside and increases linearly to 10 at the center.

What is the mass of this object? (You don't need to simplify the numbers. Remember the formula for mass: $M = \iint \delta(x, y)dA$ where δ is the density.)

7. Find the volume of the following region: all points simultaneously below the cone $z = \sqrt{x^2 + y^2}$, above the xy-plane, and inside the cylinder $(x - \frac{5}{2})^2 + y^2 = 25/4$.



8. On the line $x + y = 8$, where does the function $f(x, y) = x^3y^5$ achieve its maximum?

9. Find and classify all the critical points of $f(x, y) = 5x^3 + 6x^2y - 2y^3 - 27x$.

____ **10.** Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle y, x^2 \rangle$ and C is the curve going from $(0,0)$ to $(2,8)$ along $y = 2x^2$.

a. 8/3
b. 16/3
c. 32/3
d. 56/3
e. 25/3
f. 64/3

____ **11.** Let $\mathbf{F} = \langle z^2 \sin y, xz^2 \cos y, 2xz \sin y \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is any curve from $(3, \pi/2, 2)$ to $(6, \pi/6, 1)$.

a. 2
b. -3
c. 4
d. 6
e. 8
f. -9
g. 12
h. None of the above

____ **12.** Calculate

$$\int_C (e^x - y^3)dx + (\cos y + x^3)dy$$

where C is $x^2 + y^2 = 4$ in the counterclockwise direction.

a. $3\pi/2$
b. 24π
c. 16π
d. $5\pi/2$
e. 8π
f. $\pi/2$

____ **13.** Compute the outward flux of $\mathbf{F}$ across S if

$$\mathbf{F} = x^4\mathbf{i} + (-x^3z^2)\mathbf{j} + 4xy^2z\mathbf{k}$$

and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = x + 5$ and $z = 0$.

a. $\pi/2$
b. $\pi/6$
c. $\pi/3$
d. π
e. $\pi/4$
f. $2\pi/3$