Final Exam Math 114 Fall 2015

There are 9 problems on this exam, answer all of them. They are all multiple choice, and the first of these consists of ten true/false statements. For the true/false questions, **circle the ENTIRE word "true" or "false" as the case may be**, and for the other multiple choice questions, **circle the ENTIRE phrase you deem correct among the choices given**. There is partial credit given for the answers to the questions numbered 2 through 9 –**you must show your work to get any credit for these**. (No need to show your work for the true/false questions.) Write answers to ALL questions on the exam paper and show your computations. There are no blue books just the exam sheets. Each of the true/false questions are worth 4 points, and each of the other 8 multiple choice questions are worth 20 points, for a total of 200 points.

No books, tables, notes, calculators, computers (of any sort), cell-phones or any other electronic gear are allowed. One $8.5'' \times 11''$ page of notes (both sides OK), in your own handwriting, is allowed.

Please fill in the data below NOW.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

Your name (printed) ______________________________.

Your signature ______________________________ Date ________________

Circle Instructor's name: Chai Haglund Powers Shatz

Your Penn ID number ______________________________.

PLEASE DO NOT WRITE BELOW THIS LINE

1	6
2	7
3	8
4	9
5	TOTAL

Throughout the exam, we write f_x for $\frac{\partial f}{\partial x}$ etc.

1. True/False (4 points each answer) no partial credit, no need to show work :

a) If $\vec{v}, \vec{w}$ are vectors, each of length 1, and if $(\vec{v} \cdot \vec{w}) = 0$ then $\vec{v} + \vec{w}$ has length 2.

True False

b) If the torsion of a curve, $\vec{r}(t)$, vanishes everywhere then it must lie in a fixed plane.

True False.

c) If $\vec{v}, \vec{w}$ and $\vec{z}$ are vectors with $\vec{v} \neq \vec{0}$ and we have $\vec{v} \times (\vec{w} - \vec{z}) = \vec{0}$ and $\vec{v} \cdot (\vec{w} - \vec{z}) = 0$ then $\vec{w} = \vec{z}$.

True False.

d) Suppose $\vec{F}$ is a smooth vector field on a region containing a smooth, *closed* surface Σ. Then $\int\int_\Sigma \overrightarrow{curl F} \cdot \hat{n}\, d\sigma = 0$.

True False.

e) Let $\vec{r}(t)$ describe the path of a projectile moving only under the influence of gravity. If we know $\vec{r}(0)$ and $\overrightarrow{r'(0)}$ we can predict $\vec{r}(t)$ for $t > 0$.

True False.

f) If $f(x, y, z)$ is a smooth function on a region, R, of 3-dimensional space and if $\int\int\int_R f(x, y, z)\, dV = 0$, then $f(x, y, z) = 0$ throughout R.

True False.

g) Suppose (x_0, y_0) is a point inside a region, R, and $f(x, y)$ is a function defined on R. If $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$ at (x_0, y_0), then (x_0, y_0) is either a maximum or a minimum for $f(x, y)$ in R.

True False.

h) If $\vec{r}(t)$ is a curve in 3-dimensional space then $\overrightarrow{r'(t)}$ is a vector of length 1.

True False.

i) For any smooth function, f, on a region, the product $(div(\overrightarrow{grad(f)})(\overrightarrow{curl}(\overrightarrow{grad(f)})$ is $\vec{0}$.

True False.

j) If R is a region in the plane with a smooth boundary, ∂R, we can find a vector field, $\vec{F}$, in R so that $\int_{\partial R} \vec{F} \cdot \overrightarrow{\mathrm{d}r} =$ area of R.

True False.

Space for all computations and reasons for the true/false questions below.

More space for computations below

REMEMBER: You must show work in the next eight problems to receive any credit. A sketch of the geometry in a problem gets partial credit. 20 points total for each problem.

2. The temperature on an ellipsoid whose equation is

$$x^2 + y^2 + 2z^2 = 3$$

is given by $T(x, y, z) = xyz - 2\sqrt{2}$. Find the *sum* of the maximum and the minimum temperatures on this ellipsoid.

A) $\sqrt{2}$

B) $-2\sqrt{2}$

C) $3\sqrt{2}$

D) $-4\sqrt{2}$

E) $5\sqrt{2}$.

F) $-6\sqrt{2}$

G) $7\sqrt{2}$

3. Assume $u \leq v$. Find the maximum value of the integral

$$\int_u^v (4 - 2x - 6x^2)\, dx.$$

A) $(\frac{1}{3})^3$

B) $(\frac{1}{2})^3$

C) 1

D) $(\frac{3}{2})^3$

E) $(\frac{4}{3})^3$.

F) $(\frac{5}{3})^3$

G) 8

4. A projectile moves in the space with constant acceleration $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Suppose that its initial position when $t = 0$ is $(0, 0, 0)$, and its inital velocity when $t = 0$ is $2\mathbf{i} + \mathbf{j} - \mathbf{k}$. What is the position of this projectile when $t = 4$?

A) $(4, 0, 3)$

B) $(16, -4, 12)$

C) $(16, 10, -2)$

D) $(8, -4, 2)$

E) $(10, 6, -4)$

F) $(14, -8, 6)$

G) $(32, 10, 12)$

5. Let C be the curve

$$C = \{(x, y) \in \mathbb{R}^2 \mid x \in \mathbb{R},\ y = \frac{1}{4}(e^{2x} + e^{-2x})\}$$

Let $\kappa(x)$ be the curvature of C at $(x, \frac{1}{4}(e^{2x} + e^{-2x})$. Determine the limit

$$\lim_{x \to \infty} \kappa(x).$$

A) 0

B) 1

C) 2

D) $\frac{1}{2}$

E) 4

F) $\frac{1}{4}$

G) ∞

6. Compute the line integral

$$I = \oint_C \left(\sin(x) + z \right) dx + (e^y + x + z)dy + 2ydz$$

where C is closed contour consisting of three straight line segments from $(2, 0, 0)$ to $(0, 3, 0)$ to $(0, 0, 4)$ and back to $(2, 0, 0)$.

A) -1

B) 0

C) 4

D) 9

E) 13

F) 16

G) 20

7. Compute the flux of the force field $\overrightarrow{F}(x, y, z) = \langle xy + \sin(y), z^2 - y^2, yz + 2\rangle$ through the part of the surface $x^2 + y^2 + z = 10$ which is above the plane $z = 6$, where the normal points upward (toward increasing z).

A) 6π

B) 8π

C) 10π

D) 11π

E) 13π

F) 15π

G) 18π

8. Evaluate the integral

$$\int\int_R e^{\frac{x+y}{x-y}}\,dA,$$

where R is the trapezoidal region in the xy-plane with vertices $(1,0)$, $(2,0)$, $(0,-2)$ and $(0,-1)$.

A) 1

B) 2

C) $\frac{3}{4}(e - e^{-1})$

D) e

E) π

F) $3\sqrt{2} - e$

G) $2e$

9. Find the y-coordinate of the center of mass of the solid tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and the plane $x + y + z = 1$. Assume the mass density is $\delta(x, y, z) = y$.

A) $\frac{1}{5}$
B) $\frac{1}{4}$
C) $\frac{1}{3}$
D) $\frac{2}{5}$
E) $\frac{\sqrt{2}}{2}$
F) $\frac{1}{\sqrt{3}}$
G) $\frac{e}{6}$

The next pages give further space for computations for problems 2 through 9.

Computational Space

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