Math 114
Final Exam Fall 2016

First and Last Name ______________________________(PRINT) Penn ID__________

Professor **(circle one):** **Hilburn-001** **Zhu-002**

Rimmer-003 **Alexandersson-004**

This exam has 12 multiple choice questions, each question is worth 10 points for a total of 120 points. Partial credit will be given for the entire exam so be sure to show all work. Circle the correct answer and give supporting work, **a correct answer with little or no supporting work will receive little or no credit**. Use the space provided to show all work. A sheet of scrap paper is provided at the end of the exam. If you write on the back of any page please indicate this in some way.

You have **120 minutes** to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave.

Once you have completed the exam, sign the academic integrity statement below.
Do **NOT** write in the grid below. It is for grading purposes only.

Problem	Points	Problem	Points
1		7	
2		8	
3		9	
4		10	
5		11	
6		12	
Total			

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.

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Name (printed)

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Signature

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Date

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1. Determine k such that the volume of the parallelepiped spanned by the vectors $\langle 6,k,0\rangle, \langle 3,4,-k\rangle$, and $\langle 1,2,2\rangle$ is maximized.

A) 1 B) 2 C) 3 D) 4

E) 5 F) 6 G) 7 H) None of these

2. Find a vector $\mathbf{v}$ such that

a) $\mathbf{v}$ is orthogonal to both $\langle 1,0,1\rangle$ and $\langle 1,2,-1\rangle$

AND

b) $\mathbf{v}$ together with $\langle 0,1,-1\rangle$ spans a parallelogram with area $5\sqrt{6}$.

The magnitude of $\mathbf{v}$ is

A) $2\sqrt{5}$	B) $5\sqrt{2}$	C) $3\sqrt{5}$	D) $5\sqrt{3}$
E) $4\sqrt{2}$	F) $3\sqrt{2}$	G) $4\sqrt{3}$	H) None of these

3. Let

$$\mathbf{r}(t) = \left\langle t^2 \sin(2t), t^2 \cos(2t), \sqrt{3}\, t^2 \right\rangle.$$

Find the arclength of $\mathbf{r}(t)$ for $-2 \le t \le 2$.

A) $\dfrac{64\sqrt{2}}{3}$ B) $\dfrac{16\sqrt{2}}{3}$ C) $\dfrac{32\sqrt{2}}{3}$ D) $\dfrac{16}{3}\left(\sqrt{2}-1\right)$

E) $\dfrac{12}{3}\left(4\sqrt{2}-2\right)$ F) $\dfrac{32}{3}\left(2\sqrt{2}-1\right)$ G) $\dfrac{8}{3}\left(4\sqrt{2}-1\right)$ H) None of these

4. Let

$$f(x,y) = x^3 - y^3 - 2xy + 6$$

What is the value of $f(x,y)$ at its local maximum?

A) 0 B) 6 C) 9 D) $\frac{170}{27}$

E) $\frac{-2}{3}$ F) $\frac{55}{32}$ G) $\frac{62}{9}$ H) None of these

5. Let

$$f(x, y) = x^2 + 3y^2 + 2y$$

Find the sum of the absolute maximum value of $f(x, y)$ and the absolute minimum value of f on the unit disk $x^2 + y^2 \le 1$.

A) $\frac{2}{3}$ B) 5 C) 1 D) $\frac{1}{6}$

E) $\frac{14}{3}$ F) $\frac{13}{4}$ G) $\frac{-1}{6}$ H) None of these

6. Evaluate

$$\int_1^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2}\,dy\,dx$$

A) 2 B) $\frac{1}{2}$ C) 1 D) π

E) $\frac{1}{\pi}$ F) $\frac{\pi}{4}+\frac{1}{2}$ G) $\frac{\pi}{2}+1$ H) None of these

7. Evaluate

$$\int_0^1 \int_0^{1-x^2} \frac{2xe^{2y}}{1-y}\,dydx$$

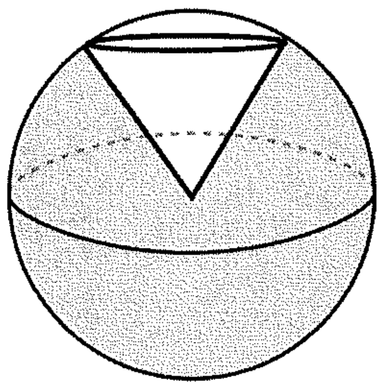
A) $2e^2$ B) $\sqrt{e}-1$ C) $\frac{e-1}{2}$ D) $\frac{e^2-1}{2}$

E) e F) $\frac{\sqrt{e}-1}{2}$ G) $\sqrt{e}$ H) Not computable

8. Compute the volume of a solid bounded below by the sphere $x^2 + y^2 + z^2 = a^2$ and above by the cone $z = \sqrt{x^2 + y^2}$. See the shaded region of the diagram below.

A) $\dfrac{2\sqrt{2}\pi a^3}{3}$ B) $\dfrac{2\pi}{3}\left(2a^3 - \tfrac{\sqrt{2}}{2}\right)$ C) $\dfrac{2\pi a^3}{3}\left(1 + \tfrac{\sqrt{2}}{2}\right)$ D) $\dfrac{\sqrt{2}\pi a^3}{3}$

E) $\dfrac{2\pi a^3}{3}\left(2 - \tfrac{\sqrt{2}}{2}\right)$ F) $\sqrt{2}\pi a^3$ G) $a^3\left(\tfrac{\pi\sqrt{2}}{2} + 1\right)$ H) None of these



9. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where

$\mathbf{F}=\left\langle \frac{z}{x},\frac{z}{y},\ln(xy)\right\rangle$ and C is the path $\mathbf{r}=\left\langle e^t,e^{2t},t^2\right\rangle$, $1\le t\le 3$

A) 60 B) 64 C) 72 D) 75
E) 78 F) 81 G) 84 H) None of these

10. Green's theorem for area states that the region R whose boundary is a simple closed curve C satisfies

$$\text{Area of } R = \frac{1}{2}\oint_C xdy - ydx.$$

Find the area of the region enclosed by one arc of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ $0 \le t \le 2\pi$ and the x-axis.

A) a^2 B) πa^2 C) $2\pi a^2$ D) $3\pi a^2$

E) $6\pi a^2$ F) $8\pi a^2$ G) $12\pi a^2$ H) None of these

11. Determine the value of the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F} = \left(\sin x - \frac{y^3}{3} \right) \mathbf{i} + \left(\cos y + \frac{x^3}{3} \right) \mathbf{j} + (xyz)\mathbf{k}$$

and C is the intersection between the cone $z^2 = x^2 + y^2$ and the plane $z = 1$ traversed counter-clockwise around the $z-$axis.

A) $\frac{\pi}{3}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

E) $\frac{4\pi}{3}$ F) $\frac{2\pi}{3}$ G) π H) None of these

12. Use the Divergence Theorem to evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{F} = \langle x + \cos y, \ln z, x^2 \rangle$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = 1$ with $z > 0$ and $\mathbf{n}$ is the outward normal to S.

A) $\frac{7\pi}{6}$ B) $\frac{11\pi}{12}$ C) $\frac{5\pi}{4}$ D) $\frac{5\pi}{12}$

E) $\frac{7\pi}{3}$ F) $\frac{5\pi}{2}$ G) $\sqrt{2}\pi$ H) None of these

Scrap Paper

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