Name: ______________________________ **PennID**: ______________

Math 114
Final Exam
May 2, 2017

Instructions:

Turn off and put away your cell phone.
Please write your Name and PennID on the top of this page.
Please sign and date the pledge below to comply with the Code of Academic Integrity.
No calculators or any other electronic devices are allowed during this exam.
The only consultation material you may have access to is a 1 page (2-sided) cheat sheet, which will be collected at the end of the exam. You may not consult any other references.
If any question is unclear, raise your hand to ask for clarifications.

In all problems, show all of your work; no credit will be given for unsupported answers.
Please try to be as organized, objective, and logical as possible in your answers.

#	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination.

______________________________ ______________

Signature Date

Problem 1 (10 pts): Consider the curve

$$\vec{r}(t) = (\sin t, t, \cos t).$$

What is the area of the triangle with vertices $\vec{r}(0)$, $\vec{r}(\pi/2)$ and $\vec{r}(\pi)$?

Problem 2 (10 pts): Find the arc length of the portion of the curve

$$\vec{r}(t) = (t^2, t\cos t, t\sin t)$$

for $0 \leq t \leq 1$.

Problem 3 (10 pts): A projectile is launched at a height of 10 feet from the ground, at an angle of $\pi/4$, and with initial speed of $48\sqrt{2}$ feet/sec. How many seconds does it take the projectile to reach a height of 42 feet (from the ground) for the first time? Take the gravitational acceleration g to be 32 feet/sec^2.

Problem 4 (10 pts): Consider the function

$$f(x,y) = \begin{cases} 4x\left(\sin\dfrac{1}{x}\right) e^{x^2y-5y^3} & \text{if } \ x \neq 0 \\ e^{-5y^3} & \text{if } \ x = 0 \end{cases}$$

a) Compute $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ if it exists.

b) Is $f(x,y)$ continuous at (0,0)?

Problem 5 (10 pts): Find the extrema (local minima, local maxima and saddle points) of the function

$$f(x, y) = 2x^3 - 3y^3 + 6xy^2 - 150x$$

Problem 6 (10 pts): Find the extrema of the function $f(x, y, z) = x + y - z$ on the curve defined by the equations

$$4x^2 + y^2 = 20, \qquad 2y = z.$$

Problem 7 (10 pts): Find the total area of the portions of the sphere $x^2+y^2+z^2=4$ that lie inside the cylinder $x^2+y^2=1$.

Problem 8 (10 pts): Use Stokes' Theorem to compute the line integral $\int_\gamma \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z) = (y, z, x)$ and γ is the intersection of the sphere $x^2 + y^2 + z^2 = a^2$, where $a > 0$, with the plane $x+y+z = 0$, oriented so that its projection on the xy-plane is oriented counterclockwise.

Problem 9 (10 pts): Use Gauss' Theorem to compute the flux $\iint_\Sigma \vec{F} \cdot \vec{n}\, d\sigma$ where $\vec{F}(x, y, z) = (x^3y^2z^2 + z^2 \cos y, 1 + x^2y^3z^2 - 2z, 4e^x + x^2y^2z^3)$, Σ is the boundary surface of the cube with vertices $(\pm 1, \pm 1, \pm 1)$, and $\vec{n}$ is the outward pointing unit normal.

Problem 10 (10 pts): Is the vector field $\vec{F}(x, y, z) = (x+z, -y-z, x-y)$ conservative on the domain $\Omega = \mathbb{R}^3$? If so, then find a potential $\varphi\colon \mathbb{R}^3 \to \mathbb{R}$ such that $\vec{F} = \nabla\varphi$.