Math 114

Final Exam
May 5, 2016.

Name: ______________________________ Penn ID #: ____________________

Instructor/section (circle/tick one): Subhrajit Bhattacharya (001): T/Th   Shilin Yu (003): MWF

Name of TA: ______________________________ Recitation day and hour: ________________

Instructions

1. Write your full name and Penn ID card number clearly in the space provided above, and **circle your section/instructor**. Please write your name as it appears on your Penn ID card. Also, write the name of your TA and day/hour of your recitation.
2. You are allowed to keep and consult a single page (US letter size page) of self-written notes containing formulae/equations/definitions/etc during the exam. Please attach/include this page with your exam submission.
3. **Calculators, books, class notes, laptops or cellphones are NOT allowed** during the exam. Please remove everything from your desk except for the single-page self-written notes mentioned above and the equipments to write with (pen/pencil, eraser and ruler).
4. You have about 2 hours to finish the exam.
5. There are 15 questions (make sure you do not miss any).
6. You need to show complete work/solution for each problem. **Just selecting the correct option/answer without showing the work will not fetch you any point.**
7. If you get stuck at a question, do not waste time on it. Move on to the next. You will be given partial credit for partially solving a problem.
8. Use the space provided below each problem to solve the problem. There are three additional pages at the end of the questions for rough/additional work. This space should be enough. However, if you really need an extra sheet of blank paper, you can get one from the proctor. Do not forget to write your name on the extra sheet and attach it to your submission.
9. Once you have completed the exam, **please sign the academic integrity statement below.**

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.

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Signature

Do not write anything in the table below. It is for grading purpose only.

Problem #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		Total
Points																	

Problem 1. Suppose the tempearture at a point (x,y,z) is

$$T(x,y,z) = \ln(x+2y+5z).$$

Find the **unit** direction $\mathbf{u}$ in which the temperature increase most rapidly at $P=(1,5,-2)$ and the maximum rate of change D_{max} of the temperature at the same point.

(A) $\mathbf{u} = \frac{1}{5}\langle 1,5,-2\rangle, D_{max} = 5$

(B) $\mathbf{u} = \frac{1}{\sqrt{30}}\langle 1,5,-2\rangle, D_{max} = \sqrt{30}$

(C) $\mathbf{u} = \frac{1}{\sqrt{30}}\langle 1,2,5\rangle, D_{max} = \sqrt{30}$

(D) $\mathbf{u} = \frac{1}{5}\langle 1,2,5\rangle, D_{max} = 5$

(E) $\mathbf{u} = \frac{1}{5}\langle 2,5,1\rangle, D_{max} = 5$

(F) $\mathbf{u} = \frac{1}{5}\langle 1,-2,5\rangle, D_{max} = 5$

(G) $\mathbf{u} = \frac{1}{5}\langle 5,-2,1\rangle, D_{max} = 5$

(H) $\mathbf{u} = \frac{1}{5}\langle 1,5,2\rangle, D_{max} = 5$

Problem 2. Let $f(x,y) = x^3 - 12xy + 8y^3$. Find all the critical points of f and classify each as a local minimum, a local maximum, or a saddle point.

Problem 3. Evaluate the integral

$$\int_0^1 \int_y^1 y e^{x^3} \, dxdy.$$

(A) $e-1$

(B) $\frac{1}{2}(e-1)$

(C) $\frac{1}{3}(e-1)$

(D) $\frac{1}{6}(e-1)$

(E) $1-e$

(F) $-\frac{1}{2}(e-1)$

(G) $-\frac{1}{3}(e-1)$

(H) $-\frac{1}{6}(e-1)$

Problem 4. Let **F** be the vector field

$$\mathbf{F}(x,y) = \langle e^y + 6x^3y^2,\ xe^y + 3x^4y + \pi\sin(\pi y)\rangle.$$

Show that **F** is conservative and find the potential function. Use the potential function to compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the path

$$\mathbf{r}(t) = t^4\,\mathbf{i} + t\,\mathbf{j},\quad 0 \le t \le 1,$$

and circle out the right answer below.

(A) e

(B) $e - \frac{1}{2}$

(C) $e + \frac{1}{2}$

(D) $-e - \frac{1}{2}$

(E) $-e + \frac{1}{2}$

(F) $-e$

(G) $e + \frac{7}{2}$

(H) $-e + \frac{7}{2}$

Problem 5. The region D in $\mathbb{R}^3$ is bounded by the planes

$$x = 0, \quad y = 0, \quad z = 0, \quad \text{and} \quad 2x + 2y + z = 4.$$

Express the triple integral

$$\iiint_D e^z dV$$

as an iterated integral with the order of integration $dzdydx$. **Evaluate the resulting integral** and circle out the right answer below.

(A) 1	(E) $\frac{1}{8}(e^2 - 5)$
(B) $\frac{1}{4}(e^4 - 13)$	(F) $\frac{1}{8}(e^2 - 13)$
(C) $\frac{1}{4}(e^4 - 5)$	(G) $\frac{1}{8}e^2$
(D) $\frac{1}{4}e^4$	(H) 42

Extra space for work:

Problem 6. Find the volume of the solid region below the paraboloid $z = x^2 + y^2 + 1$, above the coordinate plane $z = 0$, and within the cylinder $x^2 + y^2 = 9$.

(A) π

(B) 44π

(C) 40π

(D) 50π

(E) $\frac{101}{2}\pi$

(F) $\frac{99}{2}\pi$

(G) $\frac{97}{2}\pi$

(H) $-\pi$

Problem 7. Let

$$\mathbf{F}(x,y,z) = -y^2\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}.$$

Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the boundary of the part of the plane $3x + 2y + z = 6$ that lies in the first octant. Assume that C is oriented counterclockwise when viewed from above the xy-plane.

(A) -6	(E) 6
(B) -7	(F) 7
(C) -8	(G) 8
(D) -9	(H) 9

Problem 8. A projectile fired over a horizontal ground hits the ground at a distance of 20 m after 2 s. What is the launch angle of the projectile? (Assume acceleration due to gravity, $g = 10\,\text{m/s}^2$).

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

(E) $\sin^{-1}\frac{1}{\sqrt{3}}$

(F) $\sin^{-1}\frac{2}{5}$

(G) $\cos^{-1}\frac{1}{\sqrt{3}}$

(H) $\cos^{-1}\frac{2}{5}$

Problem 9. The function $z = f(x,y)$ is described implicitly by the equation

$$e^{xy} + e^{yz} + e^{zx} - 3z^3 = 0.$$

Compute $\nabla f|_{(0,0)}$.

(A) $\mathbf{i} - \mathbf{j}$

(B) $\frac{1}{\sqrt{2}}\,\mathbf{i} + \frac{1}{\sqrt{2}}\,\mathbf{j}$

(C) $\frac{1}{8}\mathbf{i}$

(D) $\frac{\sqrt{3}}{2}\,\mathbf{i} - \frac{1}{2}\,\mathbf{j}$

(E) $\frac{1}{9}\,\mathbf{i} + \frac{1}{9}\,\mathbf{j}$

(F) $\frac{1}{6}\,\mathbf{i} + \frac{1}{6}\,\mathbf{j}$

(G) $\frac{1}{\sqrt{2}}\,\mathbf{i} - \frac{1}{\sqrt{2}}\,\mathbf{j}$

(H) f is not differentiable at $(0,0)$.

Problem 10. Find the product of the minimum and maximum distance from the ellipsoid $x^2 + y^2 + 4z^2 = 4$ to the plane $x + y + z - 8 = 0$. (That is, the minimum/maximum possible distance from any point on the ellipsoid to any point on the plane.)

(A) -20

(B) 0

(C) 40

(D) $\frac{40}{3}$

(E) $\frac{55}{\sqrt{3}}$

(F) 55

(G) $\frac{55}{3}$

(H) 3

Problem 11. Using Divergence Theorem, compute the flux of the vector field

$$\mathbf{F} = (y^2 + 2x)\mathbf{i} + (z^2 - y)\mathbf{j} + (xy + z)\mathbf{k}$$

across the surface of the sphere $x^2 + 4x + y^2 + z^2 = 1$, which is oriented outward.

(A) $\frac{20(\sqrt{5}+1)}{3}\pi$

(B) $\frac{40\sqrt{5}}{3}\pi$

(C) $5\sqrt{5}\pi$

(D) $9\sqrt{5}\pi$

(E) $\frac{32}{\sqrt{5}}\pi$

(F) $\frac{20\sqrt{5}}{3}\pi$

(G) $\frac{4}{3}\pi$

(H) 0

Problem 12. Is the function

$$f(x,y) = \begin{cases} \dfrac{x^2+y^2-2}{x+y-2}, & x+y \neq 2 \\ 1, & \text{otherwise.} \end{cases}$$

continuous at $(1,1)$? Give clear explanation for your answer.

Problem 13. Let R be the region on the plane bounded by the curve described by the polar equation

$$r = 2 + \cos 2\theta, \quad 0 \leq \theta < 2\pi$$

Sketch the region R and compute its area. [Hint: $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$.]

(A) $\frac{3}{2}$

(B) $\frac{16}{3}$

(C) 4π

(D) π

(E) 9π

(F) $\frac{9}{2}\pi$

(G) $\frac{7}{3}\pi$

(H) 16

Problem 14. Using Green's Theorem, compute the circulation of the planar vector field

$$\mathbf{F} = \left(\sin(e^{\sqrt{x}}) - x^2 y\right) \mathbf{i} + \left(\sin^5 y + xy^2\right) \mathbf{j}$$

along the counterclockwise oriented unit circle centered at the origin.

(A) 0

(B) $\cos(e) + 1$

(C) $\frac{\pi}{2}$

(D) 2π

(E) 1

(F) $\frac{\pi}{4}$

(G) $\cos(e) + \frac{1}{5}$

(H) π

Problem 15. Compute the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$.

(A) 2

(B) $\frac{3}{2}\pi$

(C) 1

(D) $\frac{5\sqrt{5}}{6}\pi$

(E) $4\sqrt{2}$

(F) π

(G) 2π

(H) $\frac{5\sqrt{5}-1)}{6}\pi$

You can use this page for rough / additional work. This page will not be checked when grading, unless you clearly request the grader to do that in the space provided below a problem.

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