

Math 260 Detailed Syllabus.

Determinants:

1. Axioms for a determinant function; uniqueness, existence.
2. Properties of the determinants: elementary functions and determinants, Gauss reduction and determinants; the determinant of the inverse of a nonsingular matrix, determinants and independence of vectors, the determinant of a block-diagonal matrix, expansion formulas for determinants, minors and cofactors, the determinant of a transpose, the cofactor matrix, Cramer's rule.

Eigenvalues and eigenvectors:

1. Review: Linear transformation and Matrices.
2. Eigenvectors and eigenvalues of a linear transformation, linear independence of eigenvectors corresponding to distinct eigenvalues,.
3. Finite-dimensional case, characteristic polynomials, calculation of eigenvalues and eigenvectors in the finite-dimensional case, non diagonalizable matrices.
4. Trace of a matrix, matrices representing the same linear transformation, similar matrices.

Euclidean space; Eigenvalues of operators acting on Euclidean Space.

1. Euclidean space.
2. Hermitian, symmetric, skew-Hermitian anti-symmetric, transformations; eigenvalues and eigenvectors of Hermitian, symmetric and skew-Hermitian (antisymmetric) operators, orthogonality of eigenvectors corresponding to distinct eigenvalues, , matrix representations for Hermitian and skew-Hermitian operators, Hermitian and skew-Hermitian matrices, the adjoint of a matrix. Orthogonal transformations; properties of orthogonal transformations, orthogonal matrices, special orthogonal matrices; properties of eigenvalues of orthogonal and special orthogonal matrices.
3. The spectral theorem; the spectral theorem in matrix form.
4. Applications, in particular to L_2 spaces, and Sturm-Liouville operators.

Basic Topology

Standard topology in \mathbb{R}^n , open balls. open and closed sets ; interior and boundary of a set; bounded, compact, connected, simply connected set in \mathbb{R}^n . Product Topology and standard topology.

Calculus of multivalued functions of several variables.

Functions from \mathbb{R}^n to \mathbb{R}^m . Continuity. The total derivative and differentiability. Properties of the total derivative: partial derivatives, directional derivatives, partial derivatives of higher order, gradients, and related theorems. Chain rules (in matrix form) and product rules. Applications to geometry, level sets, tangent planes, derivatives of vector fields.

Applications of the differential Calculus to single valued functions of several variables

Positive definite, negative definite and indefinite bilinear forms and matrices. Second-order Taylor formula for scalar fields, the Hessian matrix, Extremal values, maxima, minima, saddle points. Extrema with constraints, Lagrange's multipliers method.

Integration over compact n-dimensional sets in \mathbb{R}^n .

Definition, upper and lower double integrals, and existence theorem. "Fubini's theorems" (evaluation of a multiple integral by iterated one-dimensional integrations), geometric interpretation of the integral as a volume. Properties: integrability of continuous functions, integrability of bounded functions with discontinuities.

Change of variables in multiple integrals.

Integration over other regions:

1. Line Integrals: parametrized curves, Path and line integrals, other notations for line integrals, basic properties of line integrals, the concept of work as a line integral, line integrals with respect to arc length, further applications of line integrals. Fundamental Theorem of Calculus for Line Integrals, independence of the path, necessary and sufficient conditions for a vector field to be a gradient, necessary conditions for a vector field to be a gradient.

2.a Green's theorem in the plane, some applications of Green's theorem, and necessary and sufficient condition for a two-dimensional vector field to be a gradient.

2.b Surface integrals: parametrized surfaces, area of a parametric surface, surface integrals, , other notations for surface integrals. Orientation, the theorem of Stokes, the curl and divergence of a vector field, further properties of the curl and divergence.

3. Divergence theorem (Gauss' theorem), applications of the divergence theorem.

Ordinary differential equations

Characteristic equation, initial conditions, boundary conditions, Wronskian, existence and uniqueness theorems. Homogeneous ode. Ode with constant coefficients. Regular points, series solutions. Applications and examples.

Matrix resolution of system of differential equations.

Applications and examples.