Name: ______________________________ Section (circle one): 001 002

Midterm Exam I for Math 110, Fall 2015

October 02, 2015

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	8	
7	16	
8	12	
9	8	
10	16	
Total	120	

- You have ninety minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: ______________________

Date: ______________________

1. What is the (approximate) relation between x and y if $\ln y = 4.6 + 2 \ln x$?

2. Graph the function xe^x. Please choose scales on the x- and y-axes that are neither too small nor too large to show the shape, and point out any maxima, minima and asymptotes, as well as exact coordinates of a few points on the graph.

3. Jake sums the reciprocals of the square roots of the integers from 1 to 100. Laura sums the reciprocals of the square roots from 1 to 200.

   (a) Write expressions for Jake's sum J and Laura's sum L in $\sum$ notation.

   $$J =$$

   $$L =$$

   (b) Find a lower bound for J using integrals.

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(c) Find an upper bound for J using integrals.

(d) What simple expression approximates the ratio L/J?
The same expression should work if Jake summed the first million reciprocal square roots and Laura summed the first two million.

4. A retirement account growing at an exponential rate is expected to double in 14 years.

   (a) Write an equation for the amount in the account after t years in terms of the starting contribution, assuming no further contributions. Be sure to give units for all constants.

   (b) Using your log cheatsheet, give an approximate value for the fund after 32 years if it starts with \$700,000. This can be very approximate - we only care about the first digit.

5. In each case, write the sum (call is S) in Σ notation, then evaluate it (leave in exact form, do not use decimal approximations).

   (a) $3Z + (4Z - 13) + (5Z - 26) + \cdots + (13Z - 130)$

   (b) The infinite sum $6e^{-2} + 18e^{-4} + 54e^{-6} + \cdots$.

6. Write an equation expressing the following scenario. Be sure to define variables and constants and to give units for each.

    The number of troops necessary to patrol an area after combat units are withdrawn is proportional to the 1/2 power of the area being patrolled and inversely proportional to the length of time after the withdrawal.

7. (a) Compute $\lim_{x\to 0^+} x\ln x$.

(b) Compute $\lim\limits_{x\to\infty} \dfrac{\sqrt{2x^4+1}}{3x^2}$.

(c) True or false? $\sqrt{2x^3-1} \sim x^{3/2}$ as $x \to \infty$

(d) True or false? $(\ln x)^2 \ll x$ as $x \to \infty$

8. Compute these integrals. One is a substitution, one is by parts, one is substitution then parts.

(a) $\int x^2 \ln x \, dx$

(b) $\int_0^4 \frac{x}{\sqrt{9+x^2}} \, dx$

(c) $\int_1^4 e^{\sqrt{x}} \, dx$

9. Write a trapezoidal approximation for this integral using three trapezoids of equal length. Then evaluate the sum as an exact expression. We do not want a decimal approximation.

$$\int_0^{\pi} \frac{x\cos(x)}{\pi}$$

10. John the tax cheat owes the IRS \$100,000. At the end of each year, starting in 2015, a 5% fine is imposed on whatever he owes, after which he is forced to pay \$10,000 (via wage garnishment).

   (a) How much does he owe on January 1, 2016?

   (b) How much does he owe on January 1, 2017?

   (c) How much does he owe on January 1, 2018?

   (d) Write an expression in Sigma notation for what Joe owes on January 1, 2025.

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(e) Evaluate this expression analytically by evaluating the summation and simplifying.

(f) Find a numerical approximation to this expression by using the linearization of $\ln x$ near $x = 1$ and the approximation $e^{1/2} \approx 1.65$. [Hint: if you don't have an $e^{1/2}$ in there somewhere, you have probably made a mistake.]

TABLE 8.1 Basic integration formulas

1. $\displaystyle\int k\,dx = kx + C \qquad \text{(any number } k\text{)}$

2. $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$

3. $\displaystyle\int \frac{dx}{x} = \ln|x| + C$

4. $\displaystyle\int e^x\,dx = e^x + C$

5. $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (a > 0,\ a \neq 1)$

6. $\displaystyle\int \sin x\,dx = -\cos x + C$

7. $\displaystyle\int \cos x\,dx = \sin x + C$

8. $\displaystyle\int \sec^2 x\,dx = \tan x + C$

9. $\displaystyle\int \csc^2 x\,dx = -\cot x + C$

10. $\displaystyle\int \sec x \tan x\,dx = \sec x + C$

11. $\displaystyle\int \csc x \cot x\,dx = -\csc x + C$

12. $\displaystyle\int \tan x\,dx = \ln|\sec x| + C$

13. $\displaystyle\int \cot x\,dx = \ln|\sin x| + C$

14. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$

15. $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

16. $\displaystyle\int \sinh x\,dx = \cosh x + C$

17. $\displaystyle\int \cosh x\,dx = \sinh x + C$

18. $\displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

19. $\displaystyle\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

20. $\displaystyle\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

21. $\displaystyle\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C \qquad (a >$

22. $\displaystyle\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C \qquad (x >$

Logarithm Cheat Sheet

These values are accurate to within 1%:

$$\begin{aligned} e &\approx 2.7 \\ \ln(2) &\approx 0.7 \\ \ln(10) &\approx 2.3 \\ \log_{10}(2) &\approx 0.3 \\ \log_{10}(3) &\approx 0.48 \end{aligned}$$

Some other useful quantities to with 1%:

$$\begin{aligned} \pi &\approx \frac{22}{7} \\ \sqrt{10} &\approx \pi \\ \sqrt{2} &\approx 1.4 \\ \sqrt{1/2} &\approx 0.7 \end{aligned}$$

(ok so technically $\sqrt{2}$ is about 1.005% greater than 1.4 and 0.7 is about 1.005% less than $\sqrt{1/2}$)