Name: ____ Section (circle one): 001 002

Midterm Exam I for Math 110, Fall 2017

October 03, 2017

Problem	Points	Score
1	12	
2	12	
3	9	
4	9	
5	15	
6	12	
7	12	
8	15	
9	6	
10	9	
11	9	
Total	120	

- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: ______________________________

Date: ______________________________

1. Suppose x and y are related by $\ln x = 3 \ln y$. State in words the relation between x and y.

2. Let $f(x) = \dfrac{\sin x}{x}$.

(a) Compute $\lim_{x \to 0} f(x)$.

(b) Compute $f'(x)$.

(c) Compute $\lim_{x \to 0} f'(x)$.

(d) Sketch a graph of the function f. Your graph should reflect what you computed above, as well as qualitative features of the function as $x \to \pm\infty$.

3. Write an equation for the following scenario. Be sure to give an interpretation for all variables and constants and to give units for each.

   > The amount of energy per time required to maintain a refrigeration unit starts at a level proportional to the volume of the unit and decreases exponentially after that.

4. Find a function $g(x) = cx^p$ such that $f(x) \sim g(x)$ as $x \to \infty$ where

$$f(x) = \frac{\sqrt{4/x}}{\frac{3}{x} + \frac{5}{x^2}}.$$

You do not have to prove your answer but we should be able to see how you got it.

5. Louie agree to pay Sam \$50,000 every year for 15 years, starting January 1, 2018, after which Louie gets possession of Sam's condo. Every year, Sam puts the money in an investment account that increases by 6.9% during the course of the year.

   (a) At the end of the day on January 1, 2018, how much money is in the account?

   (b) At the end of the day on January 1, 2019, how much money is in the account?

   (c) Let M be the amount of money in the account at the end of the day on January 1, 2032, right after Sam receives the last payment. Write a summation formula for M.

(d) Write a formula for M that has no summation in it.

(e) Use your log cheatsheet, linearization, and any other techniques you can think of to give a numerical estimate for the value of M. Within 10% is accurate enough.

6. In each case, write the sum (call is S) in Σ notation, then evaluate it (leave in exact form, do not use decimal approximations).

(a) $e^a + e^{2a} + e^{3a} + \cdots + e^{13a}$

(b) $\dfrac{2y+z}{x} + \dfrac{2y+2z}{x} + \cdots + \dfrac{2y+14z}{x}$

(c) The infinite sum $\dfrac{1}{2} + \dfrac{1}{8} + \dfrac{1}{32} + \cdots$

7. Use integrals to give upper and lower bounds for this sum.

$$\sum_{m=16}^{99} m\sqrt{m}$$

These should be given as analytic expressions, with no integrals left unevaluated, simplified when possible. Please include a sketch on the facing page (back of problem 6) or an extra piece of paper.

Extra credit [2 points] One of these bounds does not simplify to a rational number. Use linearization to give an approximate numerical value for this expression.

8. Compute these indefinite integrals. One is a substitution, one is by parts, and one uses both of these techniques.

(a) $\int \ln(1+x)\sqrt{1+x}\,dx$

(b) $\int x^2\sqrt{4+x^3}\,dx$

(c) $\int \arctan x\,dx$

9. Let $g(x)$ be the time in hours it takes to drill a one foot diameter shaft x meters deep into a bed of limestone. State an interpretation for the inverse function g^{-1}.

10. (a) The function $\int_1^x (\ln(t))^2 \, dt$ is a function, f, of what variable?

    (b) What is f'?

    (c) Sketch a graph of f with the input variable ranging from 1 to 7. This can be a very rough sketch; just give an idea of the scale of the vertical axis and whether the function is increasing/decreasing and curved up/down.

11. Let A be the area under the graph of $y = \sqrt{1+x^3}$, above the x-axis, between the y-axis and the vertical line $x = 2$. Circle which of these numeric values most closely estimates A. If you wish to be considered for partial credit, state your reasoning, give a sketch, etc. [Hint: what kind of approximation will be most accurate if you want the computation to be brief?]

(a) 1.4

(b) 2

(c) 2.25

(d) 2.4

(e) 3.4

(f) 4

(g) 6

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C \qquad \text{(any number } k\text{)}$

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x\,dx = e^x + C$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (a > 0,\ a \neq 1)$

6. $\int \sin x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \sin x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\sin x| + C$

14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

15. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

16. $\int \sinh x\,dx = \cosh x + C$

17. $\int \cosh x\,dx = \sinh x + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C \qquad (a >$

22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C \qquad (x >$

Logarithm Cheat Sheet

These values are accurate to within 1%:

$$\begin{aligned} e &\approx 2.7 \\ \ln(2) &\approx 0.7 \\ \ln(10) &\approx 2.3 \\ \log_{10}(2) &\approx 0.3 \\ \log_{10}(3) &\approx 0.48 \end{aligned}$$

Some other useful quantities to with 1%:

$$\begin{aligned} \pi &\approx \frac{22}{7} \\ \sqrt{10} &\approx \pi \\ \sqrt{2} &\approx 1.4 \\ \sqrt{1/2} &\approx 0.7 \end{aligned}$$

(ok so technically $\sqrt{2}$ is about 1.005% greater than 1.4 and 0.7 is about 1.005% less than $\sqrt{1/2}$)