Name: ______________________________

Midterm Exam II for Math 110, Fall 2015

October 29, 2015

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
10	12	
Total	120	

- You have ninety minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: ______________________

Date: ______________________

1. (a) Find c and p such that $\dfrac{1}{3x + x^2} \sim cx^p$ as $x \to 0$.

c = ____________

p = ____________

Show your work or justify your answer.

(b) Write this integral as a limit and say whether or not it is convergent (with justification).

$$\int_0^1 \frac{dx}{3x + x^2}$$

2. Write $\displaystyle\int_0^{\infty} \frac{dx}{x-5}\, dx$ as a limit or sum of limits, then say whether it converges and why.

3. (a) Compute the value of C that makes $\dfrac{C}{\sqrt{x}}$ a probability density on $[0, 1]$.

(b) Compute the mean of this density.

4. (a) Compute the quadratic MacLaurin polynomial $P_2(x)$ for the function $f(x) = e^{x^2}\sqrt{1+x}$.

(b) What exact value does this give as an approimxation to $e^{1/4}\sqrt{3/2}$? Write the answer in the box, simplifying fractions when possible, but not using any decimal approximations:

$P_2(1/2) =$ [ ]

5. Write a differential equation for this scenario. You do not have to solve the differential equation but you must give the interpretation of all variables and constants, their units, and indicate which is the dependent and the independent variable.

   > An old growth forest shrinks due to erosion, human encroachment, disaster and many other factors. The rate per unit time at which the area of this forest decreases is proportional to the square root of the area. However, the Nature Conservancy also adds to it at the rate of one hundred thousand acres per year.

6. (a) Does this series converge? Justify your answer.

$$\sum_{n=1}^{\infty} \frac{n^2}{n!}$$

(b) Does this series converge? Justify your answer.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{3/4}}$$

7. A revenue forecast for a tecnhological innovation forecasts earnings of n^{-2} million dollars in year n.

   (a) Express the total possible earnings of this innovation (assuming you can continue marketing it forever) as an infinite series in Sigma-notation and say whether this total is finite or infinite.

   (b) **If you answered "infinite" in part(a):**

   Approximately how much should you have earned after some large number of years, M? Use an integral approximation and write your answer as cM^p.

   If you answered "finite" in part(a):

   Approximately how much value remains to be earned after M years? Use an integral approximation and write your answer as cM^p.

8. Write an initial value problem for this integral equation.

$$y(t) = 3t + \int_2^t \frac{1}{s + y(s)} \, ds \, .$$

9. Which of these equations is satisfied by the function $y(t) = (1+t)^{-2}$? Circle any, all or none: 3 points for each correctly circled or not. Answers without justification will be graded all or nothing.

(i) $y' = \dfrac{-2y}{1+t}$

(ii) $y' = -2y^{3/2}$

(iii) $y' = -2(1+t)y^2$

(iv) $y = \displaystyle\int_1^t \frac{-2y}{1+s}\,ds + \frac{1}{4}$

10. Use Euler iteration with a step size of 1 to approximate $y(3)$ where $y(t)$ is the solution to the initial value problem

$$y' = x + \sqrt{1+y}\,; \quad y(1) = 3\,.$$

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C$ (any number k)
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$
3. $\int \frac{dx}{x} = \ln|x| + C$
4. $\int e^x\,dx = e^x + C$
5. $\int a^x\,dx = \frac{a^x}{\ln a} + C$ $(a > 0,\ a \neq 1)$
6. $\int \sin x\,dx = -\cos x + C$
7. $\int \cos x\,dx = \sin x + C$
8. $\int \sec^2 x\,dx = \tan x + C$
9. $\int \csc^2 x\,dx = -\cot x + C$
10. $\int \sec x \tan x\,dx = \sec x + C$
11. $\int \csc x \cot x\,dx = -\csc x + C$
12. $\int \tan x\,dx = \ln|\sec x| + C$
13. $\int \cot x\,dx = \ln|\sin x| + C$
14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
15. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$
16. $\int \sinh x\,dx = \cosh x + C$
17. $\int \cosh x\,dx = \sinh x + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$
21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$ $(a >$
22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C$ $(x >$

Logarithm Cheat Sheet

These values are accurate to within 1%:

$$\begin{aligned} e &\approx 2.7 \\ \ln(2) &\approx 0.7 \\ \ln(10) &\approx 2.3 \\ \log_{10}(2) &\approx 0.3 \\ \log_{10}(3) &\approx 0.48 \end{aligned}$$

Some other useful quantities to with 1%:

$$\begin{aligned} \pi &\approx \frac{22}{7} \\ \sqrt{10} &\approx \pi \\ \sqrt{2} &\approx 1.4 \\ \sqrt{1/2} &\approx 0.7 \end{aligned}$$

(ok so technically $\sqrt{2}$ is about 1.005% greater than 1.4 and 0.7 is about 1.005% less than $\sqrt{1/2}$)