Name: ____

Midterm Exam III for Math 110, Fall 2015

December 03, 2015

Problem	Points	Score
1	16	
2	12	
3	16	
4	16	
5	12	
6	12	
7	16	
8	12	
9	8	
Total	120	

- You have ninety minutes for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: ________________________________

Date: ________________________________

1. (a) Sketch the slope field in the first quadrant for the differential equation

$$\frac{dy}{dt} = \frac{y^2}{1+t}.$$

(b) Find the general solution.

(c) Find the solution with $y(2) = 1$.

(d) Sktech this solution, indicating the slope at $t = 2$ and any asymptotes.

2. Compute

$$\int_R \frac{y}{1+x^2}\, dA$$

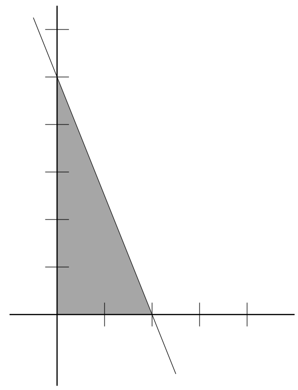
where R is the rectangle $-2 \leq y \leq 3, 1 \leq x \leq \sqrt{3}$.

3. A retirement fund grows at 3% per year (continuous growth rate) and also by the addition of \$90,000 per year (also added continuously). It starts today with no money in it.

   (a) Write an initial value problem for this. As usual, state the interpretation and units of all variables and constants.

   (b) Solve the IVP.

4. A dart lands uniformly somewhere in the triangle bounded by the x-axis, the y-axis and the line $5x + 2y = 10$.

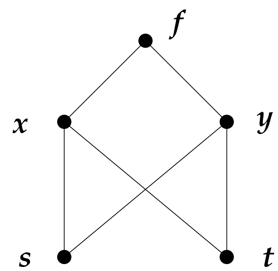


(a) Compute the probability density of the location of the dart.

(b) Compute the mean of the y-coordinate of the dart.

5. Use the increment theorem with $g(x, y) = \ln(x - \sqrt{y})$ to give a decimal approximation to $\ln(4.1 - \sqrt{8.8})$.

6. The following branch diagram refers to a function f of two variables x and y, each of which is a function of two other variables s and t.



(i) When evaluating the expression $\partial f/\partial s$, which of the variables s, t, x, y vary independently, which stay fixed, and which vary dependently?

(ii) Evaluate $\partial f/\partial s$ at the point $(s,t) = (1,4)$ if the functions in question are given by these equations:

$$\begin{aligned} f(x,y) &= \frac{x^2}{1+y} \\ x(s,t) &= e^{s+t-1} \\ y(s,t) &= s + \sqrt{t} \end{aligned}$$

7. Suppose that the quantity W of widgets that can be manufactured per unit time is a function of billions of dollars D invested in equipment and the number N of employees, via the formula

$$W = 10,000\,(D-3)\frac{N}{1500+N}\,.$$

The management is thinking of saving on annual labor cost (reducing N) by a one-time capital investment (increasing D). They need to keep production (W) constant.

How much more money would have to be invested in equipment per employee laid off if the present investment is 4.05 billion dollars and there are presently 2000 employees?

8. A function f has the following values. Estimate the value of $\frac{\partial f}{\partial x}(7, 3)$.

x	y	$f(x, y)$
7	3	10
7	5	18
6	3	12
3	7	20
70	30	100

9. Let R be the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$. Approximate

$$\int_R \sqrt{x^2 + y^2}\, dA$$

by using a Riemann sum in which the unit square is divided into four subsquares in a 2×2 grid and the function is evaluated at the center of each subsquare. Please leave the answer as an analytic expression rather than a decimal approximation.

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C \qquad \text{(any number } k\text{)}$

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x\,dx = e^x + C$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (a > 0,\ a \neq 1)$

6. $\int \sin x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \sin x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\sin x| + C$

14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

15. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

16. $\int \sinh x\,dx = \cosh x + C$

17. $\int \cosh x\,dx = \sinh x + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C \qquad (a >$

22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C \qquad (x >$