Name: ____

Midterm Exam III for Math 110, Fall 2017

December 05, 2017

Problem	Points	Score
1	18	
2	16	
3	12	
4	16	
5	14	
6	16	
7	12	
8	16	
Total	120	

- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. One 8.5×11 cheatsheet, handwritten is allowed; it may be double-sided.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: ________________________________

Date: ________________________________

1. (a) Sketch the slope field in the first quadrant for the differential equation

$$\frac{dy}{dt} = -\frac{y-3}{2}.$$

(b) On the same picture, sketch the particular solution with $y(1) = 2$.

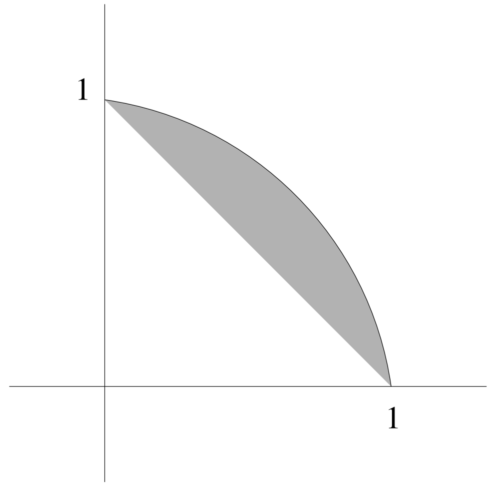
(c) Use Euler iteration with step size 1/2 to compute an approximation to $y(2)$ for this particular solution.

(d) Compute the general solution.

(e) Compute the particular solution with $y(2) = 1$.

(f) For this solution, find $\lim_{t \to \infty} y(t)$ or prove it does not exist.

2. Let R be the region inside the unit circle in the first quadrant and above the line $x + y = 1$, as shown in the figure.



(a) Describe R in the form $\{(x, y) : \quad \cdots \ \}$ using vertical strips.

(b) Suppose gold is buried in the region R and the density of gold in ounces per square meter at the point (x, y) is $\sqrt{1 - x^2} + (1 - x)$. What is the total amount of gold in the region?

3. Circle the number of the correct solution to the initial value problem

$$y' = y\frac{\sin x}{x^2}\ ; \quad y(2) = 3\,.$$

(i) $y = 3e^{2-x}$

(ii) $y = \ln\left(e^3 + \int_2^x \frac{\sin t}{t^2}\,dt\right)$

(iii) $y = e^{3+\int_2^x \frac{\sin t}{t^2}\,dt}$

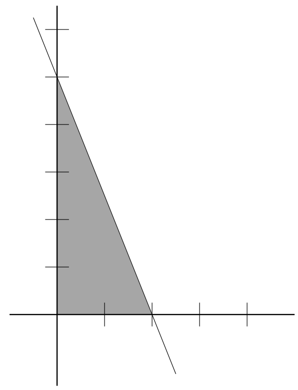
(iv) $y = 3\int \frac{\sin x}{x^2}\,dx$

(v) $y = e^{\int \frac{\sin x}{x^2}\,dx + C}$

(vi) $y = 3 + e^{\int_2^x \frac{\sin t}{t^2}\,dt}$

(vii) $y = 3e^{\int_2^x \frac{\sin t}{t^2}\,dt}$

4. A dart lands uniformly somewhere in the triangle bounded by the x-axis, the y-axis and the line $5x + 2y = 10$.



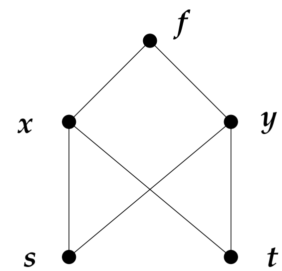
(a) Compute the probability density of the location of the dart.

(b) Compute the mean of the y-coordinate of the dart.

5. Use the increment theorem to give a decimal approximation to $\sqrt{8.9 + \ln(0.8)}$.

   Please say what you are choosing for the function $f(x, y)$ and for $x_0, y_0, \Delta x, \Delta y$ and show your computation of the partial derivatives of f at a general point before plugging in specific values.

6. The following branch diagram refers to a function f of two variables x and y, each of which is a function of two other variables s and t.



(a) Write an expression using partial derivatives of f, x and y that computes the rate of change of f per change in s when t remains constant.

(b) Evaluate this at the point $(s, t) = (1, 4)$ if the functions in question are given by these equations:

$$\begin{aligned} f(x, y) &= \frac{x^2}{1+y} \\ x(s, t) &= e^{s+t-1} \\ y(s, t) &= s + \sqrt{t} \end{aligned}$$

7. Find the two points where the curve $x^2 - xy + y^3 = 1$ crosses the x-axis. Use implicit differentiation to find the slopes of the tangents at these two points. Do the tangents intersect, or do they coincide, or are they parallel without coinciding?

8. The sales S of a lite beer, in millions, are predicted by the formula

$$S = \frac{30}{C(101 - T)}$$

where T is the taste score on a scale from 1 to 100 given by *Beer Enthusiast* magazine and C is the number of calories in a 12 oz. bottle.

(a) Compute $\dfrac{\partial S}{\partial C}$ and $\dfrac{\partial S}{\partial T}$ at the point $C = 100, T = 96$.

(b) If the brewery wants to maintain the same sales, but the taste index slips a little from 96, how must the caloric content change per point of change in taste index?

Logarithm Cheat Sheet

These values are accurate to within 1%:

$$\begin{aligned} e &\approx 2.7 \\ \ln(2) &\approx 0.7 \\ \ln(10) &\approx 2.3 \\ \log_{10}(2) &\approx 0.3 \\ \log_{10}(3) &\approx 0.48 \end{aligned}$$

Some other useful quantities to with 1%:

$$\begin{aligned} \pi &\approx \frac{22}{7} \\ \sqrt{10} &\approx \pi \\ \sqrt{2} &\approx 1.4 \\ \sqrt{1/2} &\approx 0.7 \end{aligned}$$

(ok so technically $\sqrt{2}$ is about 1.005% greater than 1.4 and 0.7 is about 1.005% less than $\sqrt{1/2}$)

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C \quad \text{(any number } k\text{)}$

2. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

3. $\int \dfrac{dx}{x} = \ln|x| + C$

4. $\int e^x\,dx = e^x + C$

5. $\int a^x\,dx = \dfrac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$

6. $\int \sin x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \sin x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\sin x| + C$

14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

15. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

16. $\int \sinh x\,dx = \cosh x + C$

17. $\int \cosh x\,dx = \sinh x + C$

18. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\dfrac{x}{a}\right) + C$

19. $\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + C$

20. $\int \dfrac{dx}{x\sqrt{x^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\left|\dfrac{x}{a}\right| + C$

21. $\int \dfrac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\dfrac{x}{a}\right) + C \quad (a >$

22. $\int \dfrac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\dfrac{x}{a}\right) + C \quad (x >$