Name: _____

Final Exam for Math 110, Fall 2015

December 17, 2015

Problem	Points	Score
1	12	
2	12	
3	16	
4	12	
5	12	
6	12	
7	20	
8	12	
9	12	
10	16	
11	12	
12	12	
Total	160	

- You have two hours for this exam.
- Please show **ALL** your work on this exam paper. Partial credit will be awarded where appropriate.
- **CLEARLY** indicate final answers. Use words (doesn't have to be that many words) to connect mathematical formulas and equations.
- **NO** books, notes, laptops, cell phones, calculators, or any other electronic devices may be used during the exam. A cheat sheet is allowed, two-pages, front and back, provided it is freshly handwritten by you.
- No form of cheating will be tolerated. You are expected to uphold the Code of Academic Integrity.

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this midterm examination.

Signature: _____

Date: _____

1. Graph the function $y = x^2 e^{-x}$. Please choose a scale that allows you to show important features such as maxima, minima, asymptotes or discontinuities.

2. Find the maximum value of the function $u(x, y) = x(1 + y^2)$ over the arc of the unit circle in the first quadrant in the xy-plane.

3. Suppose f is a function of x and y, where x is a function of the two variables s and t and y is a function of just t.

   (a) Draw a branch diagram of this.

   (b) What symbol represents "the rate of change of f per unit change in s if t is held constant starting from $s = 9$ and $t = 2$"?

   (c) Use the multivariate chain rule to write this in terms of partial derivatives. Include the information as to where the partial derivatives are evaluated.

(d) Evaluate this when

$$\begin{aligned} f(x,y) &= \frac{\sqrt{y}\ln y}{1+x} \\ x(s,t) &= s^{1-1/t} \\ y(t) &= e^t\,. \end{aligned}$$

4. Let $\mathbf{v} = 5\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and let $\mathbf{w}$ be a vector of length 3 in the 45° southeast direction.

   (a) Compute $\mathbf{v} \cdot \mathbf{v}$.

   (b) Compute $\mathbf{v} \cdot \mathbf{w}$.

   (c) Compute a unit vector $\mathbf{u}$ in the direction of $\mathbf{w}$.

   (d) Compute $\mathbf{v} \cdot \mathbf{u}$.

5. (a) For what value of C is the function Cxy a probability density on the unit square $0 \leq x, y \leq 1$?

(b) What is the mean of the X variable for this probability density?

6.

Apple is notorious for its ever-increasing cash reserves. On July 1, 2015 there were roughly 200 billion dollars. From the first of each month to the first of the next month, the existing reserves grow by one half of a percent due to diversified investment, in addition to which one billion dollars of revenue is added to the reserves.

Compute Apple's cash reserves 24 months after July 1, 2015. Please leave this as an exact expression, simplified as much as possible.

7. Let $f(x, y) = x\sqrt{16 + y^2}$.

   (a) Compute ∇f.

   (b) Evaluate this at the point $(1, 3)$, simplifying when possible.

   (c) From the point $(1, 3)$, which direction should you move in order to increase f the fastest? Answer by giving a unit vector in this direction.

(d) How fast does f increase per unit movement in this direction?

(e) At what rate does f increase per unit moved in a direction that differs from the previous direction by $45°$?

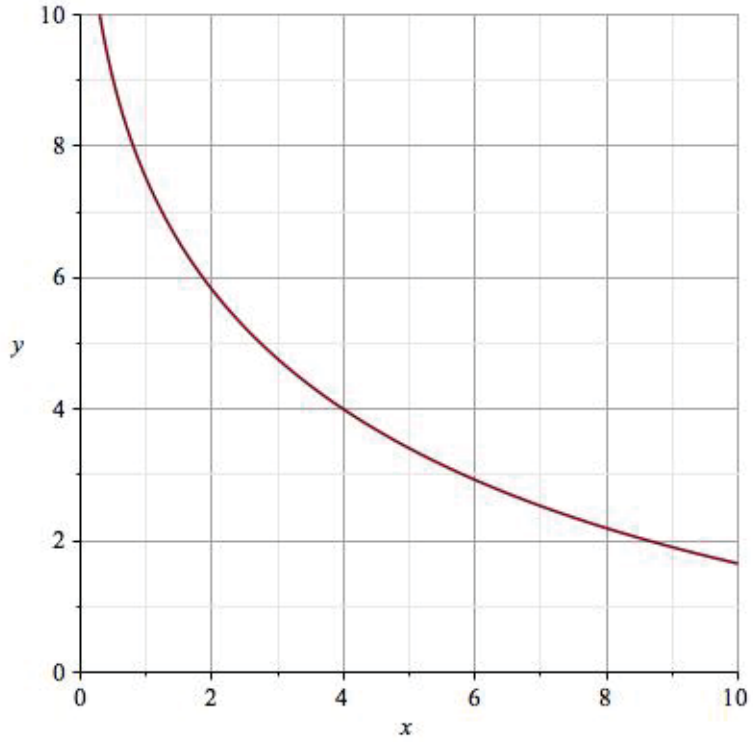
8. Use the integral test, alternating test, ratio test, root test, or the fact that the terms do not go to zero to determine if each of the following series converges or diverges. Give reasons.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \cos\left(\frac{1}{n}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2+1}$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{5^n}{n!}$

9. The figure shows a level curve of the utility function $u(x, y)$, where x is the safety rating of a motorcycle and y is its performance rating. Using the figure, mark with "F" the statements that are false, that is, are not consistent with the figure; mark with "C" the statements that are consistent with the figure.



(i) $u(7, 2.5) = u(2, 8)$.

(ii) $\nabla u(4, 4)$ points in the same direction as the vector $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$.

(iii) $\dfrac{\partial u/\partial x}{\partial u/\partial y}(7, 2.5) = 2.7$

(iv) If the safety rating is 1.0 and the performance rating is 7.5, then a small increase in safety and an equal small decrease in performance will increase utility.

10. Let $f(x) = \log_{10} x$.

(a) What is the linear Taylor approximation (linearization) for f near $x = 10$?

(b) Use this to estimate $\log_{10} 13$, leaving this as an exact expression.

(c) Is this value a lower bound or an upper bound for $\log_{10} 13$?

(d) Compute the quadratic Taylor polynomial $P_2(x)$, still at $x = 10$.

11. Sketch the region and evaluate the integral.

$$\int_0^1 \int_0^{\sqrt{x}} e^{x^{3/2}}\, dy\, dx\,.$$

12. **Growth of tech sector in developing nation with corruption:**

Suppose that available venture capital $V(t)$ has a natural continuous growth rate of 10% per year. Due to corruption, however, venture capital also leaves the sector at a rate proportional to the amount of capital and to the periodic term $2 + \cos t$ (because of the cyclic nature of anti-corruption efforts).

(a) Write a differential equation for this. Be sure to give interpretations and units for any variables or constants introduced.

(b) Find the general solution to this differential equation.

Logarithm Cheat Sheet

These values are accurate to within 1%:

$$\begin{aligned} e &\approx 2.7 \\ \ln(2) &\approx 0.7 \\ \ln(10) &\approx 2.3 \\ \log_{10}(2) &\approx 0.3 \\ \log_{10}(3) &\approx 0.48 \end{aligned}$$

Some other useful quantities to with 1%:

$$\begin{aligned} \pi &\approx \frac{22}{7} \\ \sqrt{10} &\approx \pi \\ \sqrt{2} &\approx 1.4 \\ \sqrt{1/2} &\approx 0.7 \end{aligned}$$

(ok so technically $\sqrt{2}$ is about 1.005% greater than 1.4 and 0.7 is about 1.005% less than $\sqrt{1/2}$)

TABLE 8.1 Basic integration formulas

1. $\int k\,dx = kx + C$ (any number k)

2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$

3. $\int \frac{dx}{x} = \ln|x| + C$

4. $\int e^x\,dx = e^x + C$

5. $\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad (a > 0,\ a \neq 1)$

6. $\int \sin x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \sin x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\sin x| + C$

14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

15. $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$

16. $\int \sinh x\,dx = \cosh x + C$

17. $\int \cosh x\,dx = \sinh x + C$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C \qquad (a >$

22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C \qquad (x >$