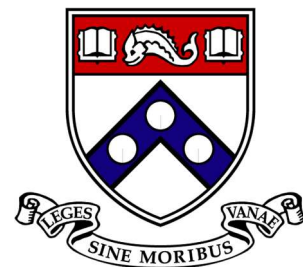


# University of Pennsylvania

## Math 104 Final Exam Spring 2017



First and Last Name \_\_\_\_\_ (PRINT) Penn ID \_\_\_\_\_

Professor (circle one): **Taylor**                      **Rimmer**                      **Stovall**

This exam has 13 questions. Each question is worth 10 points for a total of 130 points. Partial credit will be given for the entire exam so be sure to show all work. On the multiple choice, circle the correct answer and give supporting work, a correct answer with little or no supporting work will receive little or no credit. Use the space provided to show all work. Two sheets of scrap paper is provided at the end of the exam. If you write on the back of any page please indicate this in some way.

You have **120 minutes** to complete the exam. You are not allowed the use of a calculator or any other electronic device. You are allowed to use the front and back of a standard 8.5"X11" sheet of paper for handwritten notes. Please silence and put away all cell phones and other electronic devices. When you finish, please stay seated until the entire 120 minutes has elapsed. When time is up, continue to stay seated until someone comes by to collect your exam and announces that you may leave.

Do **NOT** write in the grid below. It is for grading purposes only.

Problem	Points	Problem	Points
1		8	
2		9	
3		10	
4		11	
5		12	
6		13	
7			
Total			

**My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination paper.**

\_\_\_\_\_

**Name (printed)**

\_\_\_\_\_

**Signature**

\_\_\_\_\_

1. Find the Taylor series centered at about  $x = 0$  (The Maclaurin Series) for  $f(x) = x \ln(1 + x^2)$ . Give the explicit form of the series and the first three nonzero terms.

2. Determine whether the following series is absolutely convergent, conditionally convergent or divergent.

I.  $\sum_{n=1}^{\infty} \frac{2^{3n} - 3}{11^n}$

II.  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt[3]{n+2}}$

III.  $\sum_{n=1}^{\infty} \left( \frac{n^3 + n^2 + 2}{3n^3 - n^2 + 1} \right)^{2n}$

3. Determine whether the following series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{3^{2n+1}}{2^{3n} \cdot 9^n}$$

- (A) 4            (C)  $\frac{3}{7}$             (E)  $\frac{8}{3}$             (G)  $\frac{9}{8}$
- (B)  $\frac{1}{8}$             (D)  $\frac{3}{8}$             (F) 8            (H) The series diverges

4. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+3}$$

5. Integrate

$$\int_0^1 \frac{x^2 + 3x + 1}{x^2 + 2x + 1} dx$$

(A)  $4 + \ln(4)$

(C)  $4 + \ln(2)$

(E)  $2 + \ln(4)$

(G)  $\frac{1}{2} + \ln(2)$

(B)  $\ln(2)$

(D)  $2 + \ln(2)$

(F)  $\ln(2) - \frac{1}{2}$

(H) None of these

6. Find the volume of the solid generated by revolving the region in the first quadrant bounded by  $y = 1 - x^2$ ,  $y = 0$ , and  $x = 0$  about the line  $x = 2$

(A)  $\frac{14\pi}{3}$

(C)  $\frac{17\pi}{6}$

(E)  $\frac{4\pi}{3}$

(G)  $\frac{5\pi}{6}$

(B)  $\frac{15\pi}{4}$

(D)  $\frac{13\pi}{6}$

(F)  $\frac{16\pi}{3}$

(H) None of these

7. Evaluate

$$\int_2^4 xe^{\frac{x}{2}} dx$$

(A)  $4e^2$

(C)  $\frac{1}{2}e^2$

(E)  $2e^2$

(G)  $e^2$

(B)  $\frac{3}{4}e - \frac{7}{4}e^2$

(D)  $\frac{3}{2}e - \frac{7}{2}e^2$

(F)  $\frac{e^2}{4}$

(H) None of these



8. Evaluate

$$\int \frac{\sqrt{x^2 - 9}}{x^3} dx$$

9. The function below is a probability density function

$$f(x) = \begin{cases} \frac{3}{125} x \sqrt{25 - x^2} & 0 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that  $x \geq 3$ .

- (A) 1            (C)  $\infty$             (E)  $\frac{51}{125}$             (G)  $\frac{61}{125}$
- (B)  $\frac{64}{125}$             (D)  $\frac{98}{125}$             (F)  $\frac{27}{125}$             (H) None of these

10. Evaluate

$$\int_0^{\frac{\pi}{4}} 4 \sec^4(x) \tan(x) dx$$

- (A) 1            (C) 2            (E) 3            (G) 4  
(B) 5            (D) 6            (F) 7            (H) None of these

11. Evaluate

$$\int_e^{\infty} \frac{2}{x(\ln x)^{5/3}} dx$$

- (A) 1            (C) 2            (E) 3            (G) 4  
(B) 5            (D) 6            (F) 7            (H) None of these

12. Let  $y = x\sqrt{3} + 11$ . Find the arclength for  $1 \leq x \leq 4$ .

- (A) 1            (C) 2            (E) 3            (G) 4  
(B) 5            (D) 6            (F) 7            (H) None of these

13. Solve the initial value problem

$$\frac{dy}{dx} - 2\sqrt{xy} = 0 \quad \text{with } y(1) = 4$$

Find  $y(4)$ .

- (A)  $\frac{24}{5}$       (C)  $\frac{10}{9}$       (E)  $\frac{20}{3}$       (G)  $\left(\frac{14}{3} + \sqrt{2}\right)^2$
- (B)  $\frac{2\sqrt{2}}{3}$       (D)  $\frac{4}{3}$       (F)  $\frac{400}{9}$       (H) None of these

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