1. Write a basis for the space of pairs \((u, v)\) of smooth functions \(u, v : \mathbb{R} \to \mathbb{R}\) that satisfy the system of linear differential equations

\[
\begin{align*}
    u' &= 4u - 2v \\
    v' &= 1u + 1v
\end{align*}
\]

2. Using the definition of the total derivative prove that \(\begin{bmatrix} 1 & 1 \end{bmatrix}\) is the total derivative of the function \(f(x, y) = x + y\) everywhere.

3. Determine if each of the following limits exist, if so give the limit. [You can quote any theorem stated in class.] (Prove your claims.)

1. \(\lim_{(x,y) \to (0,0)} \frac{x}{x+y}\)
2. \(\lim_{(x,y) \to (1,1)} \frac{e^{xy}}{\cos x + 3y}\)

4. Let \(V\) the vector space of polynomials of degree \(\leq 2 \in [0,1]\) with the \(L_2\) product. Let \(B = \{5, 2x, 3x^2\}\) be a basis of \(V\). Write the matrix \(Q\) corresponding to the inner product \(<,>\) with respect to the basis \(B\). Using \(Q\), compute the inner product of two arbitrary \(p, q \in V\).

5. Prove that if \(\mu\) is an eigenvalue of an orthogonal matrix (equivalently of an orthonormal transformation), then \(\mu = \pm 1\).

6. Compute the total derivative of the function \(f(x, y, z) = \frac{z}{\sqrt{x^2 + y^2}}\) at the point \((3, 4, 5)\).

7. Let \(f : \mathbb{R}^2 \to \mathbb{R}\) be defined by \(f(x, y, z) = xy + z^2\). Find all critical points of \(f\) and explain their behavior. Are there any global min/max?

8. Use Lagrange multipliers to find the extreme values of the function \(f(x, y) = x^2 + y\) along the line \(y = x\).
9. Evaluate the integral \( \int_D (x + y + z + 1)^3 \) where \( D \) is the solid bounded by the coordinate planes and the plane \( x + y + z = 1 \).
   Set up the integral in iterated form (using Fubini’s theorem) but do not complete the evaluation.

10. Let \( S \) be the surface parametrized by: \( \Phi : [1, 2] \times [0, \pi] \to \mathbb{R}^3 \) as
    \[
    \Phi(u, v) = (u \cos(v), u \sin(v), \frac{1}{2} u^2 \sin(2v)).
    \]
    Let \( f : \mathbb{R}^3 \to \mathbb{R} \) be defined by \( f(x, y, z) = e^{x+yz} \).
    Set up the complete iterated integral (using Fubini’s theorem): \( \int_S xyz \).
    Do not carry out the integration.

11. Compute the best quadratic approximation of the function \( f(x, y) = e^{x+2y} \) at the point \( (0, 0) \).
12. Let \( S \) be the surface \( \{x^2 + y^2 + z^2 = 1, \ z \geq 0\} \) and let \( \mathbf{F}(x, y, z) = (x + y + z, xy + yz + zx, xyz) \).

Use Stokes’ theorem to compute: \( \int_S \text{curl}\mathbf{F} \cdot \mathbf{n} \); here \( \mathbf{n} \) is outward normal vector. Set up the complete integral but do not carry out the integration.

13. Let \( S \) be the surface \( \{x^2 + y^2 + z^2 = 1, \ z \geq 0\} \).

Compute \( \int_S (yi + zj + xk) \cdot \mathbf{n} \) by using the divergence theorem on the solid \( \{x^2 + y^2 + z^2 \leq 1, \ z \geq 0\} \).

14. A matrix \( A \) is skew-symmetric if \( A^t = -A \) Prove that if \( A \) is a \( n \times n \) skew-symmetric matrix with \( n \) odd, then \( \det(A) = 0 \).