Math 260, Final Exam

1. Write a basis for the space of pairs (u, v) of smooth functions $u, v : \mathbb{R} \to \mathbb{R}$ that satisfy the system of linear differential equations

$$\begin{aligned} u' &= 4u - 2v \\ v' &= 1u + 1v \end{aligned}$$

2. Using the definition of the total derivative prove that $\begin{bmatrix} 1 & 1 \end{bmatrix}$ is the total derivative of the function $f(x, y) = x + y$ everywhere.

3. Determine if each of the following limits exist, if so give the limit. [You can quote any theorem stated in class.] (Prove your claims.)

1. $\lim_{(x,y)\to(0,0)}(\frac{x}{x+y})$
2. $\lim_{(x,y)\to(1,1)} \frac{e^{(xy)}}{cosx+3y}$

4. Let V the vector space of polynomials of degree $\leq 2 \in \ [0, 1]\}$ with the L_2 product. Let $\mathbf{B} = \{5, 2x, 3x^2\}$ be a basis of V. Write the matrix Q corresponding to the inner product $<,>$ with respect to the basis $\mathbf{B}$. Using Q, compute the inner product of two arbitrary $p, q \in V$.

5. Prove that if μ is an eigenvalue of an orthogonal matrix (equivalently of an orthonomal transformation) , then $\mu = \pm 1$.

6. Compute the total derivative of the function $f(x, y, z) = \frac{z}{\sqrt{x^2+y^2}}$ at the point $(3, 4, 5)$.

7. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y, z) = xy + z^2$. Find all critical points of f and explain their behavior. Are there any global min/max?

8. Use Lagrange multipliers to find the extreme values of the function $f(x, y) = x^2 + y$ along the line $y = x$.

9. Evaluate the integral $\int_D (x+y+z+1)^3$ where D is the solid bounded by the coordinate planes and the plane $x+y+z=1$.
Set up the integral in iterated form (using Fubini's theorem) but do not complete the evaluation.

10. Let S be the surface parametrized by: $\Phi : [1,2] \times [0,\pi] \longrightarrow \mathbb{R}^3$ as

$$\Phi(u,v) = (u\cos(v),\ u\sin(v),\ \frac{1}{2}u^2\sin(2v)\ ).$$

Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined by $f(x,y,z) = e^{x+yz}$.

Set up the *complete* iterated integral (using Fubini's theorem): $\int_S xyz$.
Do not carry out the integration.

11. Compute the best quadratic approximation of the function $f(x,y) = e^{x+2y}$ at the point $(0,0)$.

12. Let S be the surface $\{x^2 + y^2 + z^2 = 1,\ z \geq 0\}$ and let $\mathbf{F}(x, y, z) = (x + y + z, xy + yz + zx, xyz)$.

Use Stokes' theorem to compute: $\int_S curl\mathbf{F} \cdot \mathbf{n}$; here $\mathbf{n}$ is outward normal vector. Set up the *complete* integral but do not carry out the integration.

13. Let S be the surface $\{x^2 + y^2 + z^2 = 1,\ z \geq 0\}$.
Compute $\int_S (y\mathbf{i} + z\mathbf{j} + x\mathbf{k}) \cdot \mathbf{n}$ by using the divergence theorem on the solid $\{x^2 + y^2 + z^2 \leq 1,\ z \geq 0\}$.

14. A matrix A is skew-symmetric if $A^t = -A$ Prove that if A is a $n \times n$ skew-symmetric matrix with n odd, then $\det(A) = 0$.