1. Let \( F(x, y) = (2x \cos(2y), -2x^2 \sin(2y)) \). Compute \( \int_C F \cdot dr \) between the points \( P = (0, 0) \) and \( Q = (1, \pi/2) \) in two different ways:

1. By parametrizing a curve \( C \) (of your choice) between \( P \) and \( Q \).
2. Using the fundamental theorem of calculus for line integrals.

2. Let \( S \) be the surface parametrized by: \( \Phi : [1, 2] \times [0, \pi] \to \mathbb{R}^3 \)
\[
\Phi(u, v) = (u \cos(v), u \sin(v), \frac{1}{2} u^2 \sin(2v))
\]
Let \( f : \mathbb{R}^3 \to \mathbb{R} \) be defined by \( f(x, y, z) = xyz \).
Set up the complete iterated integral (using Fubini’s theorem): \( \int_S xyz \).
Do not carry out the integration.

3. Find the volume of the region in the first octant cut out by the cylinder \( x^2 + y^2 = a^2 \) and the planes \( x = y, y = 0, z = 0 \).
Set up the complete iterated integral (using Fubini’s theorem).
Do not carry out the integration.

4. Using Green’s theorem evaluate the integral \( \oint_D F \cdot dr \) where:
\( F(x, y) = (x \cos y, x^2 \sin y) \) and \( D = \{ (x, y) \text{ s. t. } x \geq 0, 1 + x^2 \leq y \leq 2 \} \).
Set up the complete iterated integral (using Fubini’s theorem).
Do not carry out the integration.

5. Prove or disprove the following statement: Stokes’ theorem can be applied to any surface \( S \subset \mathbb{R}^3 \) with smooth boundary \( \partial S \).

6. Using Stokes’ theorem compute \( \int_S F \cdot n \) where:
\( F(x, y) = (7x, 0, -z) \), \( S \) is the boundary of the solid sphere of radius 2 centered at the origin and \( n \) the outward pointing normal vector.

7. 1. Prove that the function \( f(x, y) = \sqrt{x^2 + y^2} \) is not differentiable at the point \( (0, 0) \).
2. Prove that the function \( f(x, y) = \sqrt{x^2 + y^2} \) is differentiable at the point \( (1, 1) \).

8. Let \( f : \mathbb{R}^2 \to \mathbb{R} \) be defined by \( f(x, y) = x^2 + 2xy + 2y^2 + x + 2y + 2 \).
Find all critical points of \( f \) and explain their behavior. Are there any global min/max?
9. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \frac{1}{(x^3 + y)^2 + (y^3 + 1)^2}.$$ 

Compute the total derivative by using the function the chain rule.

10. Which of the following functions can be defined continuously at $P = (0, 0)$, for some suitable $b \in \mathbb{R}$? Prove your answers. Find the value of $b$.

1. $f(x, y) = \begin{cases} \frac{x - y}{x + y} & \text{if } (x, y) \neq (0, 0) \\ b & \text{if } (x, y) = (0, 0) \end{cases}$

2. $f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^3 + y^2} & \text{if } (x, y) \neq (0, 0) \\ b & \text{if } (x, y) = (0, 0) \end{cases}$

3. $f(x, y) = \begin{cases} \frac{x^4 - y^2}{\sqrt{(x^2 + y^2)}} & \text{if } (x, y) \neq (0, 0) \\ b & \text{if } (x, y) = (0, 0) \end{cases}$