Math 260, Sample Final Exam

1. Let $\mathbf{F}(x,y) = (2x\cos(2y), -2x^2\sin(2y))$. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ between the points $P = (0,0)$ and $Q = (1, \pi/2)$ in two different ways:

1. By parametrizing a curve C (of your choice) between P and Q.
2. Using the fundamental theorem of calculus for line integrals.

2. Let S be the surface parametrized by: $\Phi : [1,2]\times[0,\pi] \to \mathbb{R}^3$ as

$$\Phi(u,v) = (u\cos(v),\ u\sin(v),\ \frac{1}{2}u^2\sin(2v)\ ).$$

Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined by $f(x,y,z) = xyz$.

Set up the *complete* iterated integral (using Fubini's theorem): $\int_S xyz$.
Do not carry out the integration.

3. Find the volume of the region in the first octant cut out by the cylinder $x^2 + y^2 = a^2$ and the planes $x = y$, $y = 0$, $z = 0$.

Set up the *complete* iterated integral (using Fubini's theorem).
Do not carry out the integration.

4. Using Green's theorem evaluate the integral $\oint_{\partial D} \mathbf{F}\cdot d\mathbf{r}$ where:
$\mathbf{F}(x,y) = (x\cos y, x^2\sin y)$ and $D = \{(x,y) \text{ s. t. } x \geq 0,\ 1 + x^2 \leq y \leq 2\}$.

Set up the complete iterated integral (using Fubini's theorem).
Do not carry out the integration.

5. Prove or disprove the following statement: Stokes' theorem can be applied to any surface $S \subset \mathbb{R}^3$ with smooth boundary ∂S.

6. Using Stokes' theorem compute $\int_S \mathbf{F}\cdot\mathbf{n}$ where:
$\mathbf{F}(x,y) = (7x, 0, -z)$, S is the boundary of the solid sphere of radius 2 centered at the origin and $\mathbf{n}$ the outward pointing normal vector.

7.
1. Prove that the function $f(x,y) = \sqrt{(x^2+y^2)}$ is not differentiable at the point $(0,0)$.
2. Prove that the function $f(x,y) = \sqrt{(x^2+y^2)}$ is differentiable at the point $(1,1)$.

8. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = x^2 + 2xy + 2y^2 + x + 2y + 2$. Find all critical points of f and explain their behavior. Are there any global min/max?

9. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \frac{1}{(x^3+y)^2+(y^3+1)^2}.$$

Compute the total derivative by using the function the chain rule.

10. Which of the following functions can be defined continuously at $P = (0,0)$, for some suitable $b \in \mathbb{R}$? Prove your answers. Find the value of b.

1. $f(x,y) = \begin{cases} \frac{x-y}{x+y} & \text{if } (x,y) \neq (0,0) \\ b & \text{if } (x,y) = (0,0) \end{cases}$
2. $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^3+y^2} & \text{if } (x,y) \neq (0,0) \\ b & \text{if } (x,y) = (0,0) \end{cases}$
3. $f(x,y) = \begin{cases} \frac{x^4-y^2}{\sqrt{(x^2+y^2)}} & \text{if } (x,y) \neq (0,0) \\ b & \text{if } (x,y) = (0,0) \end{cases}$