Math 103  
Name_____________________________  
Final Exam  
December 21, 2009  
Student ID # ____________________

Circle the name of your professor :  
Sesum  
Rimmer  
Wylie  

Circle your recitation:  
Seshamani  
Shahshahani  
Chase  
Vergadia  
Kimmel  

There are ten multiple choice questions (7 points each) and four free response questions (8 points each). Carefully transfer your multiple choice answers to this sheet. Show all work. A correct answer with little or no credit will receive no credit. You are allowed the use of a standard 8.5” X 11” sheet of notes. You are not allowed to use a calculator. Please put away and silence all cell phones and other electronic devices.

1. (A) (B) (C) (D) (E) (F) (G) (H)  
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8. (A) (B) (C) (D) (E) (F) (G) (H)  
9. (A) (B) (C) (D) (E) (F) (G) (H)  
10. (A) (B) (C) (D) (E) (F) (G) (H)  

For Grading Purpose Only

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1. If \( \int_{0}^{3} f(x) \, dx = 12 \) and \( \int_{0}^{6} f(x) \, dx = 42 \), find the value of \( \int_{3}^{6} (2f(x) - 3) \, dx \).

   a) 50   b) 51   c) 52   d) 56

   e) 53   f) 54   g) 55   h) None of these

2. Find the value of the integral \( \int_{0}^{8} \sqrt[3]{x} \left( \frac{1}{\sqrt{x^2}} + 1 \right) \, dx \).

   a) 14   b) 6   c) 12   d) 20

   e) 18   f) 72   g) \frac{5}{2}   h) 168
3. Find the value of \( \int_{e}^{2} \frac{(\ln x)^2}{x} \, dx \).

- a) \ln 2
- b) \frac{1}{2} \ln 2
- c) \frac{1}{2}
- d) \frac{3}{2}
- e) 1
- f) \frac{1}{\ln 2}
- g) 0
- h) \frac{7}{3}

4. Find the value of \( \int_{0}^{\ln 9} e^{x/2} \, dx \).

- a) 4
- b) 2
- c) 6
- d) 3
- e) 16
- f) 9
- g) 8
- h) 1
5. Find all critical numbers for the function \( f(x) = \sqrt[3]{9-x^2} \).

a) 0  b) -3  c) 0, -3  d) No critical numbers
e) 3  f) 3, -3  g) 0, 3, -3  h) None of these

6. At what value(s) of \( x \) is the function \( f(x) = \begin{cases} x^2 + 4x + 5 & \text{if } x < -2 \\ \frac{1}{2}x & \text{if } -2 \leq x \leq 2 \\ 1 + \sqrt{x-2} & \text{if } x > 2 \end{cases} \) discontinuous?

a) -2  b) 0  c) -2, 0, and 2  d) -2 and 0
e) 2  f) -2 and 2  g) 0 and 2  h) \( f \) is continuous everywhere
7. Find the interval on which the graph of \( f(x) = \ln(x^2 + 1) \) is concave upward.

   a) (-1,1)  b) (-1,2)  c) (-2,1)  d) (-2,2)

   e) (-1,3)  f) (-3,2)  g) (-3,3)  h) \((-\infty, \infty)\)

8. The curve \( y = x^3 + x^2 - x \) has two horizontal tangents. Find the distance between these two horizontal lines.

   a) \(\frac{11}{9}\)  b) \(\frac{22}{27}\)  c) \(\frac{32}{27}\)  d) \(\frac{5}{3}\)

   e) \(\frac{14}{9}\)  f) \(\frac{4}{3}\)  g) \(\frac{13}{9}\)  h) \(\frac{7}{3}\)
9. If \( f(x) = \frac{x}{\tan x} \), find \( f'\left(\frac{\pi}{4}\right) \).

\[
\begin{align*}
\text{a) } & \frac{2-\pi}{2} & \text{b) } & \frac{1-\pi}{2} & \text{c) } & 1-\pi & \text{d) } & \frac{\pi}{2} \\
\text{e) } & 1-2\pi & \text{f) } & 2-\pi & \text{g) } & 2-2\pi & \text{h) } & -\pi
\end{align*}
\]

10. Evaluate the limit \( \lim_{x \to \infty} \frac{\ln(3+2e^{5x})}{6x} \).

\[
\begin{align*}
\text{a) } & 5 & \text{b) } & \frac{1}{5} & \text{c) } & \frac{5}{3} & \text{d) } & \frac{3}{5} \\
\text{e) } & \frac{1}{6} & \text{f) } & \frac{1}{10} & \text{g) } & 10 & \text{h) } & \frac{5}{6}
\end{align*}
\]
11. Let $V$ be the volume of a cylinder having height $h$ and radius $r$, and assume that $h$ and $r$ vary with time. When the height is 5 in. and is increasing at 0.2 in./s., the radius is 3 in. and is decreasing at 0.1 in./s. How fast is the volume changing at that instant?

12. A rectangle with base on the $x$-axis has its upper vertices on the curve $y = 12 - x^2$. Find the maximum area of such a rectangle. Be sure to prove that you have found the maximum area.
13. The graph of \( f \) below consists of line segments and semicircles. Let \( g(x) = \int_0^x f(t) \, dt \).

Answer the following questions.

(a) \( g(14) \)

(b) \( g(10) \)

(c) \( g'(6) \)

(d) What is the absolute minimum value of \( g \) on the interval \([0,14]\)?
14. Given the graph of \( y = f'(x) \), answer the questions that follow.

(a) Find all values of \( x \) at which (Explain your answers for full credit)

(i) \( f \) is increasing.

(ii) \( f \) is decreasing.

(iii) \( f''(x) > 0 \).

(iv) \( f \) has an inflection point.

(v) \( f \) has a local maximum

(b) Sketch a graph which could represent \( y = f(x) \).