Math 103
Final Exam
Fall 2009

1. If \( \int_{0}^{3} f(x) \, dx = 12 \) and \( \int_{0}^{6} f(x) \, dx = 42 \), find the value of \( \int_{3}^{6} \left( 2f(x) - 3 \right) \, dx \).

   a) 50   b) 51   c) 52   d) 56
   e) 53   f) 54   g) 55   h) None of these

2. Find the value of the integral \( \int_{0}^{8} \left( x^2 - \frac{1}{\sqrt[4]{x^2}} + 1 \right) \, dx \).

   a) 14   b) 6   c) 12   d) 20
   e) 18   f) 72   g) \frac{5}{2}   h) 168

3. Find the value of \( \int_{e}^{e^2} \left( \frac{\ln x}{x} \right)^2 \, dx \).

   a) \ln 2   b) \frac{1}{2} \ln 2   c) \frac{1}{2}   d) \frac{3}{2}
   e) 1   f) \frac{1}{\ln 2}   g) 0   h) \frac{7}{3}

4. Find the value of \( \int_{0}^{\ln 9} e^{x/2} \, dx \).

   a) 4   b) 2   c) 6   d) 3
   e) 16   f) 9   g) 8   h) 1

5. Find all critical numbers for the function \( f(x) = \sqrt{9 - x^2} \).

   a) 0   b) -3   c) 0, -3   d) No critical numbers
   e) 3   f) 3, -3   g) 0, 3, -3   h) None of these
6. At what value(s) of \( x \) is the function \( f(x) = \begin{cases} x^2 + 4x + 5 & \text{if } x < -2 \\ \frac{1}{2}x & \text{if } -2 \leq x \leq 2 \\ 1 + \sqrt{x-2} & \text{if } x > 2 \end{cases} \) discontinuous?
   
a) -2 b) 0 c) -2, 0, and 2 d) -2 and 0
   e) 2 f) -2 and 2 g) 0 and 2 h) \( f \) is continuous everywhere

7. Find the interval on which the graph of \( f(x) = \ln(x^2 + 1) \) is concave upward.
   
a) (-1, 1) b) (-1, 2) c) (-2, 1) d) (-2, 2)
   e) (-1, 3) f) (-3, 2) g) (-3, 3) h) \((-\infty, \infty)\)

8. The curve \( y = x^3 + x^2 - x \) has two horizontal tangents. Find the distance between these two horizontal lines.
   
a) \( \frac{11}{9} \) b) \( \frac{22}{27} \) c) \( \frac{32}{27} \) d) \( \frac{5}{3} \)
   e) \( \frac{14}{9} \) f) \( \frac{4}{3} \) g) \( \frac{13}{9} \) h) \( \frac{7}{3} \)

9. If \( f(x) = \frac{x}{\tan x} \), find \( f'\left(\frac{\pi}{4}\right) \).
   
a) \( \frac{2 - \pi}{2} \) b) \( \frac{1 - \pi}{2} \) c) \( 1 - \pi \) d) \( \frac{\pi}{2} \)
   e) \( 1 - 2\pi \) f) \( 2 - \pi \) g) \( 2 - 2\pi \) h) \( -\pi \)

10. Evaluate the limit \( \lim_{x \to \infty} \frac{\ln(3 + 2e^{5x})}{6x} \).
    
a) 5 b) \( \frac{1}{5} \) c) \( \frac{5}{3} \) d) \( \frac{3}{5} \)
    e) \( \frac{1}{6} \) f) \( \frac{1}{10} \) g) 10 h) \( \frac{5}{6} \)
11. Let \( V \) be the volume of a cylinder having height \( h \) and radius \( r \), and assume that \( h \) and \( r \) vary with time. When the height is 5 in. and is increasing at 0.2 in./s., the radius is 3 in. and is decreasing at 0.1 in./s. How fast is the volume changing at that instant?

12. A rectangle with base on the \( x \)-axis has its upper vertices on the curve \( y = 12 - x^2 \). Find the maximum area of such a rectangle. Be sure to prove that you have found the maximum area.

13. The graph of \( f \) below consists of line segments and semicircles. Let \( g(x) = \int_0^x f(t)\,dt \).

Answer the following questions.

(a) \( g(14) \)

(b) \( g(10) \)

(c) \( g'(6) \)

(d) What is the absolute minimum value of \( g \) on the interval \([0,14]\)?
14. Given the graph of $y = f'(x)$, answer the questions that follow.

(a) Find all values of $x$ at which (Explain your answers for full credit)
   (i) $f$ is increasing.
   (ii) $f$ is decreasing.
   (iii) $f''(x) > 0$.
   (iv) $f$ has an inflection point.
   (v) $f$ has a local maximum

(b) Sketch a graph which could represent $y = f(x)$. 
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Answers

1. B
2. E
3. H
4. A
5. G
6. A
7. A
8. C
9. A
10. H

11. $\frac{-6\pi}{5}$

12. 32 $un^2$

13. a) $4 + \frac{5\pi}{2}$  b) $12 + \frac{9\pi}{2}$  c) 2  d) 0

14. a) i) $(x_1, x_3)$  ii) $(x_0, x_1) \cup (x_3, x_4)$  iii) $(x_0, x_2)$  iv) $x_2$  v) $x_3$

b)