1. Solve the initial-value problem. \( \frac{dx}{dt} + 2tx = x, \ x(0) = 5 \). Use your solution to compute \( x(3) \).
   
   a) \( 5e^{-6} \)  
   b) \( 5e^6 \)  
   c) \( 6e^5 \)  
   d) \( 3 \)  
   e) \( -10 \)  
   Ans: a

2. The volume of the solid generated by revolving the region bounded by the curves \( x = y^2 \) and \( y = x - 2 \) about the \( y \)-axis
   
   a) \( \frac{20\pi}{3} \)  
   b) \( \frac{72\pi}{5} \)  
   c) \( \frac{42\pi}{5} \)  
   d) \( \frac{13\pi}{2} \)  
   e) \( \frac{32\pi}{5} \)  
   f) \( \frac{212\pi}{15} \)  
   Ans b

3. Find the volume of the solid generated by rotating about the \( y \)-axis the region enclosed by \( y = \sin x \) and the \( x \)-axis from \( x = 0 \) to \( x = \pi \).

   (A) \( \frac{\pi^2}{2} \)  
   (B) \( \frac{\pi}{2} \)  
   (C) 4  
   (D) 2  
   (E) \( 4\pi^2 \)  
   (F) \( 2\pi^2 \)  

4. Which of the following statements is true about the series \( \sum_{n=0}^{\infty} (-1)^n \cos \left( \frac{1}{n} \right) \)? Be sure the work you show justifies your choice.

   a) the series is absolutely convergent  
   b) the series is conditionally convergent  
   c) the series is divergent  
   Ans: c

5. What is the interval of convergence of the series \( \sum_{n=0}^{\infty} \frac{n^3 x^{3n}}{n^4 + 1} \)?

   a) the series converges only at \( x = 0 \)  
   b) the series converges for all \( x \)  
   c) the series diverges for \( x \neq 0 \)  
   d) the series converges on \((-1, 1]\)  
   e) the series converges on \([-1, 1)\)  
   f) the series converges on \([-1, 1]\)
6. Find the Taylor series about \( a = 0 \) for \( \frac{1}{1+2x^2} \).

   (A) \( \sum_{n=0}^{\infty} (-1)^n \cdot 2^{2n} \cdot x^{2n} = 1 - 4 \cdot x^2 + 16 \cdot x^4 - 64 \cdot x^6 + \ldots \)
   (B) \( \sum_{n=0}^{\infty} (-1)^n \cdot 2^n \cdot x^{2n} = 1 - 2 \cdot x^2 + 4 \cdot x^4 - 8 \cdot x^6 + \ldots \)
   (C) \( \sum_{n=0}^{\infty} 2^{2n} \cdot x^{2n} = 1 + 4 \cdot x^2 + 16 \cdot x^4 + 64 \cdot x^6 + \ldots \)
   (D) \( \sum_{n=0}^{\infty} 2^n \cdot x^{2n} = 1 + 2 \cdot x^2 + 4 \cdot x^4 + 8 \cdot x^6 + \ldots \)
   (E) \( \sum_{n=0}^{\infty} x^{2n} = 1 + x^2 + x^4 + x^6 + \ldots \)
   (F) \( \sum_{n=0}^{\infty} (-1)^n x^{2n} = 1 - x^2 + x^4 - x^6 + \ldots \)

   Ans: b

7. What is the length of the part of the curve \( y = x^2 - \frac{\ln x}{8} \) between the points \((1,1)\) and \((e^2, e^2 - \frac{1}{8})\)?

   (a) \( y = e^3 - \frac{1}{2} \)  
   (b) \( y = \frac{1}{2} e^3 - \frac{3}{8} \)  
   (c) \( y = \frac{1}{3} e^2 + \frac{1}{8} \)  
   (d) \( y = e^2 - \frac{7}{8} \)  
   (e) \( y = \frac{1}{2} e^2 - \frac{3}{8} \)  
   (f) \( y = \frac{1}{3} e^3 - \frac{5}{8} \)

   Ans d

8. Integrate: \( \int_{0}^{1} \frac{3x+2}{x^2 - 4} \, dx \)

   (A) \(-2\)  
   (B) \(-\ln 3\)  
   (C) \(-\ln 2\)  
   (D) \(0\)  
   (E) \(\ln 2\)  
   (F) \(2\)

   a) \(-2\)  
   b) \(-3\ln 2 + \ln 3\)  
   c) \(\ln 2\)  
   d) \(\pi/4\)  
   e) \(0\)  
   f) \(\ln(3)\)

   Ans: b
Consider the region $S$ bounded by the curves $y = x + \frac{1}{x^2}$ and $y = x - \frac{1}{x^2}$ for $x \geq 1$.

Is the area $S$ finite or infinite? If finite, what is the area?

a) the integral diverges, so the area is not finite  
b) area = 0  
c) area = 1  
d) area = 2  
e) area = $\pi$  
f) area = 4  

Ans. d

10. Evaluate the following integral: $\int_0^4 \frac{3\ln 4x}{\sqrt{x}} \, dx$

   a) $12\ln 2 - 12$  
b) 0  
c) $12(1 - \ln 2)$  
d) 0  
e) 12  
f) divergent  

Ans a

11. Evaluate the integral or show it is divergent: $\int_1^\infty \frac{dx}{x\ln x}$

   a) 0  
b) 1  
c) e  
d) $e^e$  
e) $\ln(4)$  
f) divergent  

Ans: f
12. The base of a solid is the region enclosed by the ellipse \(4x^2 + y^2 = 1\). If all the plane cross-sections perpendicular to the \(x\) axis are semicircles, compute the volume of the solid.

\[
\frac{\pi}{6}, \quad \frac{\pi}{4}, \quad \frac{\pi}{3}, \quad \frac{\pi}{2}, \quad \frac{2\pi}{3}, \quad \frac{3\pi}{4}
\]

Ans c

13. Which of the following statements about the alternating series \(\sum_{n=1}^{\infty} (-1)^n a_n\) where \(a_n = \frac{n}{1 + n^2}\) is true?

a) the series is absolutely convergent
b) the series is conditionally convergent
c) the series is divergent

Ans: b

14. Evaluate the integral \(\int_0^1 \frac{x^3}{\sqrt{1 - x^2}} \, dx\)

a) \(\pi/4\) b) \(\pi/2\) c) \(\pi\) d) \(2/3\) e) \(3/4\) f) 1

Ans: d

15. Solve the differential equation. \(7yy' = 5x\)

a. \(7x^2 - 5y^2 = C\) b. \(5x^2 + 7y^2 = C\) c. \(5x^2 - 7y^2 = C\) d. \(7x^2 + 5y^2 = C\) e. \(5x^2 + 7y^2 = 12\)

Ans: c

16. Find the average value of \(f(x) = \sin^2 x \cos^3 x\) over the interval \([-\pi, \pi]\)

a) \(\pi\) b) 0 c) \(\pi/5\) d) \(\pi/6\) e) \(\pi/12\) f) \(1/2\pi\)

Ans: b

17. Consider the sequence defined by \(a_n = \frac{(-1)^n + n}{(-1)^n - n}\). Does this sequence converge and, if it does, to what limit?

a) yes, to \(-1\) b) yes, to 0 c) yes, to 1 d) yes, to 2 e) yes to \(\pi\) f) diverges

Ans: a

18. Find the area of the surface obtained by rotating the curve \(y = \frac{1}{4}x^2 - \frac{1}{2}\ln x\), \(1 \leq x \leq 2\) about the \(y\)-axis.

a. \(\frac{101\pi}{2}\) b. \(\frac{99\pi}{2}\) c. \(48\pi\) d. \(24\pi\) e. \(12\pi\) f) none of these
Ans: f
19. Use the Alternating Series Estimation Theorem or Taylor's Inequality to estimate the largest range of values of $x$ for which the given approximation is accurate to within the stated error.

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24}, \quad |\text{error}| < 0.08$$

a) only for $x = 0$  

b) $-1 \leq x \leq 1$  

c) $-\sqrt{7.2} \leq x \leq \sqrt{7.2}$  

d) $-2 \leq x \leq 2$

e) $-\sqrt{57.6} \leq x \leq \sqrt{57.6}$  

f) $-\pi \leq x \leq \pi$

Ans. e

20. Find a series representation for $\int \frac{e^x}{x} \, dx$.

a) $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)!} + C$

b) $+ C$

c) $\ln |x| + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!} + C$

d) $\ln |x| + \sum_{n=1}^{\infty} \frac{n+1}{n!} x^n + C$

e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} x^n + C$

f) $\ln |x| + \sum_{n=1}^{\infty} \frac{1}{(n + 1)!} x^{n+1} + C$

Ans: c