NAME (print):

Math 104 / Fall 2011

FINAL EXAM

Rules:
• One sheet of paper (8½ by 11 inch) both sides handwritten notes is permitted.
• No other written or printed materials are allowed.
• No electronic devices (cellular, calculator, iPad, etc.) are allowed.

Grading:
• Each problem is worth 10 points (partial credit possible).
• Do all 15(fifteen) problems, showing your work and circling your answers.
• No credit will be given for just guessing and not showing the work leading to the answer.

Instructions:
• Fill out the information requested below, and at the top of every page of this exam.
• Check that your exam booklet contains cover page + eight pages (15 problems).

Signature:

Class:

Recitation (#, day & time):
1. The value of the integral \[ \int_{-1}^{1} \left( \sqrt[3]{x} + \frac{1}{1+x^2} + \frac{1}{2-x} \right) \, dx \] is:

(A) \( 1 + \frac{\pi}{2} \)  (B) \( \frac{47}{10} \)  (C) \( \frac{\pi}{2} + \ln 3 \)  (D) \( \ln 3 + 3 \)  (E) \( 1 + 2\pi \)  (F) \( 0 \)  (G) \( \frac{\pi}{2} - \ln 3 \)  (H) \( 1 \)

2. Find the length of the arc of the curve defined by \( y = \frac{2}{3} \sqrt{x^3} \) for \( 0 \leq x \leq 3 \).

(A) \( \frac{\pi}{2} \)  (B) \( \frac{\pi}{4} \)  (C) \( 4 \)  (D) \( 5 \ln 3 \)  (E) \( \frac{14}{3} \)  (F) \( \frac{1}{4} \)  (G) \( \frac{e}{8} \)  (H) \( \frac{\ln 3}{2} \)
3. Find the volume obtained by rotating the region between the graph of \( y = \frac{1}{2} \sin^2(x^2) \) and the \( x \)-axis for \( 0 \leq x \leq \sqrt{\pi} \) about the \( y \)-axis.

(A) \( \frac{\pi}{2} \)  (B) \( \frac{\pi^2}{4} \)  (C) \( \frac{5}{4} \)  (D) \( \frac{3\pi^2}{4} \)  (E) \( \frac{1}{2} \)  (F) \( \frac{1}{4} \)  (G) \( \frac{\pi}{8} \)  (H) \( \frac{\pi^2}{8} \)

4. Evaluate \( \int_{1}^{e^3} \frac{\ln x}{\sqrt[3]{x^2}} \, dx \).

(A) \( 3e - 9 \)  (B) \( 3e^2 - 9 \)  (C) \( 9e^2 - 3 \)  (D) \( 3e^2 \)  (E) \( 9e^2 \)  (F) \( 9 \)  (G) \( 9e - 3 \)  (H) \( 3e \)
5. Find the area bounded by the $x$-axis and the graph of $y = xe^{-2x}$ for $0 \leq x < \infty$.

(A) 1  (B) 2  (C) $e - 2$  (D) $\frac{1}{4}$  (E) $\frac{1}{2}$  (F) $\frac{1}{e}$  (G) $\frac{1}{2e}$  (H) $\frac{1}{4e}$

6. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(5x - 3)^n}{n^2}$.

(A) $(-1, 1)$  (B) $[-1, 1]$  (C) $[1, \frac{4}{5}]$  (D) $[-\frac{4}{5}, \frac{4}{5}]$  (E) $[-\frac{4}{5}, \frac{4}{5}]$  (F) $[\frac{2}{5}, \frac{4}{5}]$  (G) $[0, 1]$  (H) $\{0\}$
7. Let $f(x) = e^{-x^2}$. Then $f^{(10)}(0)$ is

(A) $-\frac{1}{120}$  (B) $\frac{1}{10!}$  (C) $\frac{10}{9!}$  (D) $-\frac{10!}{9!}$  (E) $\frac{3}{10}$  (F) $\frac{1}{100}$  (G) 1  (H) 0

8. The region bounded by $y = \frac{x}{\sqrt{(x^2 + 3)^5}}$, the $x$-axis, and $0 \leq x \leq 1$, is rotated about the $x$-axis. The volume of the resulting solid is equal to:

(A) $\frac{\pi}{6}$  (B) $\frac{1}{\sqrt{2}}$  (C) $\frac{\pi}{2}$  (D) $\frac{\pi}{4}$  (E) $\frac{\pi}{2}$  (F) sec 2  (G) $\frac{1}{2}$  (H) $\frac{\pi}{72}$
9. Which of the following is the best approximation of \( \ln\left(\frac{11}{10}\right)\)?

(A) 0  (B) \(\frac{1}{10}\)  (C) \(\frac{5}{100}\)  (D) \(\frac{9}{100}\)  (E) \(\frac{95}{1000}\)  (F) \(\frac{99}{1000}\)  (G) \(\frac{109}{1000}\)  (H) \(\frac{155}{1000}\)

10. Consider the function \( f(x) = \frac{1}{x} e^{-x^2} \sin 2x \) for \( x \neq 0 \) and \( f(0) = 2 \). The order three Taylor polynomial \( a_0 + a_1 x + a_2 x^2 + a_3 x^3 \) of \( f(x) \) about \( x = 0 \) is:

(A) \(2 - \frac{10}{3} x^2\)  (B) \(2x - \frac{4}{3} x^3\)  (C) \(2 - \frac{4}{3} x^2\)  (D) \(2 - x^2\)

(E) \(x - \frac{1}{3} x^3\)  (F) \(1 + x - x^3\)  (G) \(-2 + x + \frac{10}{3} x^2\)  (H) \(2 - x + x^2\)
11. Let \( y(x) \) be the solution to the initial-value problem \( x \frac{dy}{dx} - 2y = x^3 \) and \( y(1) = 0 \). What is \( y(3) \)?

(A) 1     (B) 3     (C) 6     (D) 9     (E) 12     (F) 15     (G) 18     (H) 27

12. A random variable has as probability density function \( p(x) = 2(x + 1)^{-3} \) for \( x \geq 0 \) and \( p(x) = 0 \) else. What is the mean of the random variable?

(A) \( \sqrt{2} \)     (B) \( \frac{3}{2} \)     (C) 1     (D) \( 2\sqrt{2} \)     (E) 2     (F) 4     (G) 0     (H) \( \frac{11}{2} \)
13 Evaluate the integral \( \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan t}{\ln(\cos t)} dt \).

(A) \( \frac{1}{4e} \)  (B) \( \frac{1}{2e} - \frac{1}{e} \)  (C) \( \ln 2 \)  (D) \( 2e + \frac{2}{e} \)  (E) \( \frac{2}{e} \)  (F) \( -\ln 2 \)  (G) \( \ln 3 - 1 \)  (H) \( \frac{4}{e} \)

[Hint: \( \tan t = \frac{\sin t}{\cos t} \), etc.]

14 Which of the following series converge?

(I) \( \sum_{n=2}^{\infty} \frac{\ln n}{n^3} \)  (II) \( \sum_{n=2}^{\infty} \frac{n^3}{\ln n} \)  (III) \( \sum_{n=1}^{\infty} \frac{n}{2^n} \)  (IV) \( \sum_{n=1}^{\infty} e^{1/n} \)

(A) I & II  (B) I & III  (C) I & IV  (D) II & III  (E) II & IV  (F) III & IV  (G) all four of them  (H) none of them
15. The values of $p \geq 0$ for which the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges are precisely:

(A) $p > 1$  (B) $p > 0$  (C) $p \geq 1$  (D) $p \leq 1$  (E) $p < 1$  (F) $p > \frac{1}{2}$  (G) $p > 2$  (H) none.