Math 104 Make-up Final Exam — Spring 2010

1. Compute the total area bounded by the curves \( y = x \) and \( y = x^3 \).
   (a) 3/4  (b) 2/3  (c) 1/2  (d) 1/3  (e) 1/5  (f) 0

2. Consider the the region bounded by the curve \( y = \sqrt{x} \) and the lines \( x = 1 \) and \( y = 0 \). Find the volume of the solid obtained by rotating this region about the \( y \)-axis.
   (a) 3\(\pi/4\)  (b) \(\pi/4\)  (c) \(\pi/3\)  (d) \(\pi/5\)  (e) 3\(\pi/5\)  (f) 4\(\pi/5\)

3. Find the volume obtained by rotating the solid bounded by the curves \( y = x^2 \) and \( y = x^3 \) about the \( x \)-axis.
   (a) \(\pi/7\)  (b) 2\(\pi/7\)  (c) \(\pi/5\)  (d) 4\(\pi/5\)  (e) 2\(\pi/35\)  (f) 4\(\pi/35\)

4. Evaluate \( \int_0^1 (\pi \sin \pi x - \frac{1}{x^2 + 1}) \, dx \).
   (a) 2 - \(\pi/4\)  (b) 13\(\frac{1}{4}\) + ln 4  (c) \(e^3 - 2\frac{1}{4}\)  (d) \(\cos \frac{4}{3} - \ln 2\)  (e) 172\(\frac{1}{2}\) - \(\sqrt{3}\)  (f) 1.2146

5. Evaluate \( \int_1^e \ln x \, \frac{dx}{x^2} \).
   (a) \(e - e^{-2}\)  (b) 1 - \(\frac{2}{e}\)  (c) \(\ln \frac{e^2}{x^2}\)  (d) 1 - \(\frac{3}{e}\)  (e) \(\sqrt{\ln 2} - 1\)  (f) \(2^e\)

6. Evaluate \( \int_0^1 \frac{dx}{\sqrt{4-x^2}} \).
   (a) 0  (b) 1  (c) 1 + \(\pi\)  (d) \(\pi/6\)  (e) \(\pi/2\)  (f) \(\pi^2\)

7. Evaluate \( \int_0^2 \frac{4-2x}{(x+2)(x^2+4)} \, dx \).
   (a) \(\pi/4 - 1/2\)  (b) \(\frac{1}{2} \ln 2\)  (c) 32/51  (d) \(\pi/8\)  (e) \(e/9\)  (f) \(\cos 2\)

8. Evaluate the improper integral \( \int_0^2 \frac{1}{(x-1)^{\frac{3}{2}}} \, dx \).
   (a) \(\frac{2}{3} - \pi/2\)  (b) 1/ln 2  (c) 1.442  (d) 1 - \(e\)  (e) \(\pi - 2\)  (f) The integral is divergent.

9. Find the arclength of the part of the curve \( x = \frac{1}{3}(y^2 + 2)^{\frac{3}{2}} \) between the points \((\sqrt{3}, 1)\) and \((2\sqrt{6}, 2)\).
   (a) 10/3  (b) 7/3  (c) 2  (d) 14/3  (e) 1  (f) 5/3

10. Which of the following integrals corresponds to the surface area of revolution obtained by rotating the graph of \( y = e^x \), from \( x = 0 \) to \( x = 1 \), about the \( x \)-axis?
    (a) \(\int_0^1 \pi e^{2x} \sqrt{1 + e^{2x}} \, dx\)  (b) \(\int_0^1 \pi xe^x \sqrt{e^x + e^{2x}} \, dx\)  (c) \(\int_0^1 2\pi e^x \sqrt{1 + e^x} \, dx\)
    (d) \(\int_0^1 2\pi e^x \sqrt{1 + e^{2x}} \, dx\)  (e) \(\int_0^1 \pi e^{2x} \sqrt{e^x - 1} \, dx\)  (f) \(\int_0^1 \pi e^{x+1} \sqrt{e^x - e^{-x}} \, dx\)
11. What is the average value of the function \( f(x) = \sin x \) between \( x = 0 \) and \( x = \pi \)?
(a) 0  (b) 1  (c) 1/2  (d) \( \pi/6 \)  (e) 2/\( \pi \)  (f) 1/\( \sqrt{2} \)

12. Consider the initial value problem \( \frac{dy}{dt} = \frac{t}{y} \), with \( y(0) = -3 \). Find \( y(4) \).
(a) -3  (b) 3  (c) -5  (d) 5  (e) -7  (f) 7

13. A population is observed to obey the logistic equation \( \frac{dP}{dt} = 2P \left( 1 - \frac{P}{1000} \right) \). If \( P(0) = 500 \), when does the population reach 2000?
(a) \( t = 1 \)  (b) \( t = 2 \)  (c) \( t = 10 \)  (d) \( t = 500 \)  (e) \( t = 1000 \)  (f) never.

14. Determine if the sequence \( a_n = \left( n + e^n \right)^{1/n} \) converges or diverges. If it converges, find the limit.
(a) 1  (b) 2  (c) \( e \)  (d) 1 + \( e \)  (e) 3  (f) The sequence is divergent.

15. Determine if the series \( \sum_{n=0}^{\infty} \frac{3 \cdot 2^{n+1}}{5^n} \) converges or diverges. If it converges, find the sum.
(a) 2  (b) 3  (c) 5  (d) 10  (e) 15  (f) The series is divergent.

16. Determine all real numbers \( r \) for which the series \( \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^r + \ln n} \) converges.
(a) \( r > -1 \)  (b) \( r > 0 \)  (c) \( r > 1/2 \)  (d) \( r > 1 \)  (e) \( r > 3/2 \)  (f) \( r > 2 \)

17. Examine the two series below for absolute convergence (A), conditional convergence that is not absolute (C), or divergence (D).
(1) \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)} \)  (2) \( \sum_{n=1}^{\infty} (-1)^{n-1}2^{-n} \)
(a) 1C, 2A  (b) 1A, 2C  (c) 1A, 2D  (d) 1A, 2A  (e) 1C, 2C  (f) 1C, 2D

18. Find the interval of convergence of \( \sum_{n=0}^{\infty} \frac{x^n}{2n+1} \).
(a) [-1, 1]  (b) [-1, 1)  (c) (-1, 1)  (d) (-1, 1]  (e) (-2, 2)  (f) (-2, 2]

19. The Maclaurin series for the function \( \frac{x^2}{1-x^3} \) is
(a) \( 1 - x + x^2 - x^3 + \cdots \)  (b) \( 1 + \frac{3x^2}{2!} + \frac{4x^3}{3!} - \frac{5x^4}{4!} + \cdots \)
(c) \( x^2 - x^3 + x^4 - x^5 + \cdots \)  (d) \( x^2 + x^5 + x^8 + x^{11} + \cdots \)
(e) \( x^{2n} + x^{4n} + x^{6n} + \cdots \)  (f) \( x^3 + x^5 + x^7 + x^9 + \cdots \)

20. The coefficient of \( (x - 1)^{10} \) in the Taylor series for the function \( e^x \) at \( x = 1 \) is
(a) 1  (b) \( \frac{1}{10!} \)  (c) \( \frac{e^{10}}{10!} \)  (d) \( \frac{e^{10}}{10} \)  (e) \( \frac{e}{10!} \)  (f) \( e^{10} \).