Final Exam - Math 114 - Fall 2011

Each problem is worth 10 points. Circle your answers. In order to get full credit, you must both show your work and get the right answer.

Name: (please print) ______________________________

Circle the name of your lecturer: Cooper Haglund Pantev Powers

TA's Name: (please print) ______________________________

Day of week and time of recitation:______________________

My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this examination

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Signature

The table below is for grading purposes - do not write below this line

1.__________	8.________
2.__________	9.________
3.__________	10.________
4.__________	11.________
5.__________	12.________
6.__________	13.________
7.__________	14.________

1. Find the area of the parallelogram three of whose vertices are $(0,0,0)$, $(1,2,3)$, and $(-1,1,-1)$.

(A) $\sqrt{29}$ **(B)** $\sqrt{38}$ **(C)** $\sqrt{30}$ **(D)** 8

(E) $\sqrt{5}$ **(F)** $2\sqrt{5}$ **(G)** 6

2. Find $\mathbf{r}(t)$ if

$$\frac{d^2\mathbf{r}}{dt^2} = \langle -t^2, 1, -t \rangle$$
$$\frac{d\mathbf{r}}{dt}(1) = \langle 2/3, 0, -1/2 \rangle$$
$$\mathbf{r}(0) = \langle 1, -1, 0 \rangle$$

What is the value of $\mathbf{r}(1)$?

(A) $\langle 23/12, -3/2, -1/6 \rangle$

(B) $\langle 2, -1, 0 \rangle$

(C) $\langle 2, 1, 0 \rangle$

(D) $\langle 2 - 1, 1 \rangle$

(E) $\langle 2, 0, -1 \rangle$

(F) $\langle 3, 0, 0 \rangle$

(G) $\langle 3, -1, -1 \rangle$

3. A spring gun at ground level fires a golf ball at an angle of 45 degrees. The ball lands ten meters away. Assuming the acceleration due to gravity is 9.8 m/s^2, what is the balls initial speed?

(A) 4 m/s **(B)** 5.2 m/s **(C)** 5 m/s **(D)** $5\sqrt{2}$ m/s

(E) 10 m/s **(F)** 7.2 m/s **(G)** $\sqrt{98}$ m/s

4. Let L be the line through the origin that is perpendicular to the plane $2x + y + z = 7$. Find the distance between the point $(-4, 3, 5)$ and L.

(A) $\sqrt{2}$	**(B)** 0	**(C)** $1/5$	**(D)** $5\sqrt{2}$
(E) 10	**(F)** $\sqrt{7}$	**(G)** $2\sqrt{15}$	

5. Which of the following statements are true?

(i) The curvature of the curve $\overrightarrow{r}(t) = \langle 3\cos(t), -5 + 3\sin(t), 1\rangle$ is constant and equal to $1/2$.

(ii) If the acceleration of a motion $\overrightarrow{r}(t) = \langle x(t), y(t), z(t)\rangle$ is everywhere zero, then the trajectory of the motion is a circle.

(A) (i) only

(B) (ii) only

(C) (i) and (ii)

(D) none

6. Which of the following limits exist?

(i) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 - y^2}$

(ii) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x - y}{x^2 + y^2}$

(iii) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x + y}{\sqrt{x^2 + y^2}}$

(A) (i) only

(B) (ii) only

(C) (iii) only

(D) (i) and (ii) only

(E) (i) and (iii) only

(F) (ii) and (iii) only

(G) none

7. Find the equation of the plane that is tangent to the surface

$$\cos(y + x) - \sin(y + z) = \sin(z) - \cos(x)$$

at the point $(\pi, \pi, 0)$. What is the y-coordinate of the point where this tangent plane intersects the y-axis?

(A) 3 **(B)** π **(C)** 0

(D) $\sqrt{\pi}$ **(E)** $\sqrt{3}$ **(F)** 1

(G) $\pi/\sqrt{2}$

8. Find the product of the maximal and the minimal values of the function

$$f(x, y) = x - 2y + 2xy$$

in the region

$$(x-1)^2 + (y+1/2)^2 \leq 2.$$

(A) -1 **(B)** $-\sqrt{2}$ **(C)** 1

(D) -3 **(E)** $2\sqrt{2}$ **(F)** 0

(G) $-1/15$

9. Compute the double integral

$$\int_0^1 \int_{e^y}^{e} \frac{e-x}{\ln(x)} dx dy$$

(A) 1 **(B)** π **(C)** $\frac{(e-1)^2}{2}$

(D) $1-e$ **(E)** πe **(F)** $\frac{\pi}{2}$

(G) $\sqrt{2}$

10. Compute the line integral

$$\int_C \overrightarrow{F} \cdot d\overrightarrow{r}$$

of the vector field

$$\overrightarrow{F} = \left\langle 2xy^2 + 3xz^2, 2x^2y + 2y, 3x^2z - 2z \right\rangle$$

on the curve C given by

$$\overrightarrow{r}(t) = \langle \cos(2t) + 5\sin(5t), 6\sin(t) + 4\sin(5t), \cos(2t) + \cos(5t) \rangle$$

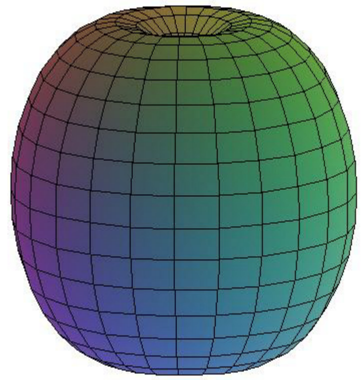
for $0 \leq t \leq \pi$.

(A) $\pi - 2$ **(B)** -2 **(C)** 3π

(D) 4 **(E)** $5 - \pi$ **(F)** 6

(G) $\cos(5)$

11. Find the volume of the solid R bounded by the surface given in spherical coordinates by the equation $\rho = (\sin\phi)^{1/3}$.



(A) 4π **(B)** $\pi - 1$ **(C)** $\frac{\pi}{3}$

(D) $\frac{4\pi}{3}$ **(E)** $\frac{\pi^2}{3}$ **(F)** $\frac{\pi}{2}$

(G) $\sqrt{2}\pi$

12. Find the value of the integral

$$\iint_R \cos\left(\frac{x-y}{x+y}\right) dA,$$

where R is the triangle in the xy-plane with vertices $(0,0)$, $(2,2)$, and $(2+\pi, 2-\pi)$.

(A) 2π **(B)** π **(C)** 4

(D) $3\pi/2$ **(E)** 3 **(F)** $5/2$

(G) -1

13. Find the y-coordinate of the center of mass of a thin plate in the shape of the upper half of the unit circle:

$$x^2 + y^2 = 1; \qquad y \geq 0$$

if the density δ at the point (x, y) is $\delta(x, y) = x^2 + y^2$.

(A) $3/4$ **(B)** $\pi^2/12$ **(C)** $1/2$

(D) $\pi/4$ **(E)** $8/(5\pi)$ **(F)** $1 - \pi/8$

(G) $2/\pi$

14. Evaluate the integral

$$\int_C \left(y + \sin\left(e^{x^2}\right)\right) dx - 2x dy,$$

where C is the circle $x^2 + y^2 = 1$ traversed counterclockwise.

(A) $\sin(e)$ **(B)** -3π **(C)** $2e$

(D) $-\pi$ **(E)** -1 **(F)** 0

(G) $\cos\left(e^{2\pi} - 1\right)$

Solution Key

1. **(B)**
2. **(A)**
3. **(G)**
4. **(D)**
5. **(D)**
6. **(A)**
7. **(B)**
8. **(D)**
9. **(C)**
10. **(B)**
11. **(E)**
12. **(C)**
13. **(E)**
14. **(B)**