MATH 114, FINAL EXAM
MAY 7th 2009, 12-2PM

Name: ____________________________  Penn ID number: __________

Signature: ____________________________

Circle the name of your instructor:

Ballard  Krieger  Powers  Zywina

Instructions:

1. The exam is 2 hours long and consists of 19 questions. Each question is worth 5 points.

2. You may use one handwritten two-sided page of notes. No other notes, books or calculators are allowed.

3. You must show all work; answers without supporting work will be given little or no credit.

The following is for grading purposes only, do not fill in.

1. ____________________________  8. ____________________________  15. ____________________________
2. ____________________________  9. ____________________________  16. ____________________________
3. ____________________________ 10. ____________________________  17. ____________________________
4. ____________________________ 11. ____________________________  18. ____________________________
5. ____________________________ 12. ____________________________  19. ____________________________
6. ____________________________ 13. ____________________________
7. ____________________________ 14. ____________________________  Total: __________
(1) Where does the plane through the points \((x, y, z) = (1, 0, 0), (0, 1, 0), (1, 1, 2)\) intersect the \(z\)-axis?
Answer: \(z = \)
A. -4 B. -2 C. -1 D. 0 E. 1 F. 2 G. 4 H. 6

(2) Suppose that \(y(x)\) satisfies the differential equation \(xy' + y = 2x\). Given the initial condition \(y(1) = 4\), what is \(y(2)\)?
A. 4 B. 7/2 C. \(2\sqrt{2}\) D. \(3\sqrt{2}\) E. \(\sqrt{3}\) F. 3 G. 5/2 H. 1

(3) Find the maximum of the function \(F(x, y, z) = 2x + y - z\) on the surface
\[4x^2 + 2y^2 + z^2 = 40.\]
Max =
A. 1 B. 2 C. 4 D. 5 E. 7 F. 10 G. 13 H. 25

(4) Find the distance between the plane through the points \((x, y, z) = (1, 0, 0), (0, 1, 0), (0, 0, -2)\), and the origin \((0, 0, 0)\)?
Distance =
A. \(\frac{1}{2}\) B. \(\frac{2}{3}\) C. \(\frac{\sqrt{2}}{2}\) D. \(\frac{4}{5}\) E. \(\frac{6}{7}\) F. 1 G. \(\frac{3}{2}\) H. 2

(5) Evaluate the integral
\[\int \int_R x + y \, dA\]
where \(R\) is the the region inside the triangle with vertices \((x, y) = (0, 0), (2, 0)\) and \((0, 1)\).
A. 0 B. 1/4 C. 1/3 D. 1/2 E. 2/3 F. 3/4 G. 1 H. 4/3

(6) The helix given by the parametric equations
\[x = 4 \sin(t) \quad y = 4 \cos(t) \quad z = 3t\]
has constant curvature. What is the curvature of this helix?
A. 0 B. \(\frac{1}{17}\) C. \(\frac{4}{25}\) D. \(\frac{1}{4}\) E. \(\frac{1}{3}\) F. \(\frac{1}{2}\) G. \(\frac{1}{\sqrt{2}}\) H. 1

(7) Find the volume of the region that is inside the sphere \(x^2 + y^2 + z^2 = 4\) and above the cone \(z = \sqrt{x^2 + y^2}\) (i.e., \(z \geq \sqrt{x^2 + y^2}\) and \(x^2 + y^2 + z^2 \leq 4\)).
A. \(\frac{8\pi}{3}\) B. \(\frac{16\pi}{3}(1 - 1/\sqrt{2})\) C. \(2\pi(\sqrt{2} - 1)\) D. \(\frac{32\pi}{3}\)
E. \(\frac{16\pi}{5}\) F. \(12\pi\) G. \(4\pi(1 - 1/\sqrt{2})\) H. \(18\pi\)
(8) Which of the following unit vectors points in the direction of fastest increase for the function \( f(x, y) = (x^2 + y^2)e^{-xy} \) at the point \((1, 0)\)?

A. \(\langle 1/\sqrt{2}, 1/\sqrt{2} \rangle\)  
B. \(\langle -3/5, 4/5 \rangle\)  
C. \(\langle 5/\sqrt{34}, 3/\sqrt{34} \rangle\)  
D. \(\langle 0, -1 \rangle\)  
E. \(\langle 2/\sqrt{5}, -1/\sqrt{5} \rangle\)  
F. \(\langle 1/2, \sqrt{3}/2 \rangle\)  
G. \(\langle -12/13, 5/13 \rangle\)  
H. \(\langle 1, 0 \rangle\)

(9) Evaluate the line integral \(\oint_C -y \, dx + x \, dy\), where \(C\) is the closed curve running counterclockwise around the circle \(x^2 + y^2 = 4\).

A. \(-5\pi\)  
B. \(-2\pi\)  
C. \(-\pi\)  
D. 0  
E. \(\pi\)  
F. \(3\pi\)  
G. \(8\pi\)  
H. \(11\pi\)

(10) Suppose \(y\) is a function of \(t\) satisfying the differential equation \(\frac{dy}{dt} = ky\), where \(k\) is a constant. Suppose \(y\) satisfies the initial conditions \(y(0) = 4\) and \(y(1) = 2\). What is \(y(2)\)?

A. 4  
B. \(4/e\)  
C. \(2/e\)  
D. \(\sqrt{2}\)  
E. \(\sqrt{5}\)  
F. 1  
G. \(1/e\)  
H. 0

(11) Find the arc length of the plane curve \(y = 2x^{3/2}\) from \(x = 0\) to \(x = 1/3\).

A. \(1/4\)  
B. \(1/3\)  
C. \(15/64\)  
D. \(14/27\)  
E. \(11/167\)  
F. \(1/2\)  
G. \(4/3\)  
H. \(\sqrt{2}\)

(12) The tangent plane to the surface \(4x^2 + 2y^2 + z^2 = 10\) at \((x, y, z) = (1, 1, 2)\) intersects the \(z\)-axis at a unique point \((0, 0, z_0)\). What is \(z_0\)?

A. \(-1\)  
B. 0  
C. 1  
D. 2  
E. 4  
F. 5  
G. 7  
H. 10

(13) A particle travels along a path \(C\) given by the vector-valued function

\[ \mathbf{r}(t) = t\sqrt{\pi/2} \mathbf{i} + t^2(1 - t^2) \mathbf{j} \]

with \(0 \leq t \leq 1\). As the particle moves along \(C\) it is subjected to a force given by the vector field

\[ \mathbf{F}(x, y) = 2x \cos(x^2 + y^2) \mathbf{i} + (2y \cos(x^2 + y^2) + 1) \mathbf{j}. \]

Find the work done on the particle by the force.

(Recall that the work is given by the line integral \(\int_C \mathbf{F} \cdot d\mathbf{r}\).)

A. \(-e^{4\pi}\)  
B. 0  
C. 1  
D. \(\frac{\pi^2}{4}\)  
E. 30  
F. \(\pi\)  
G. \(-42\)  
H. \(\ln(10)\)

(14) Evaluate the integral

\[ \int_0^2 \int_{2y}^4 e^{-x^2/2} \, dx \, dy. \]
(You may need to change the order of integration)

A. $1 - e^{-1}$  
B. $(1 - e^{-8})/2$  
C. $2(1 - e^{-8})$  
D. $1 - e^{-4}$

E. 2  
F. $(1 - e^{-16})/4$  
G. $e^2 - 1$  
H. $e^{-4}$

(15) The space curves define by the vector-valued functions

$$\mathbf{r}(t) = (t^2, \sin(t), t^4) \quad \text{and} \quad \mathbf{s}(t) = (t^3, t, \sin(t))$$

intersect at the point $(0, 0, 0)$ when $t = 0$. What is the angle (in radians) between the two curves at the point $(0, 0, 0)$.

A. 0  
B. $\pi/6$  
C. $\pi/4$  
D. $\pi/3$  
E. $\pi/2$  
F. $2\pi/3$  
G. $3\pi/4$  
H. $5\pi/6$

(16) A cylinder of solid metal is given by the region in space bounded by $x^2 + y^2 = 25$ and the planes $z = 0$ and $z = 4$. The density function of the cylinder is $\rho(x, y, z) = e^{x^2+y^2}$. What is the mass of the cylinder?

A. $4\pi(e^{25} - 1)$  
B. $8\pi$  
C. $8\pi(e^5 - 1)$  
D. $10\pi$

E. $10\pi e^{16}$  
F. $\pi(e^{10} - 1)$  
G. $\pi(e^{-25} - 1)/4$  
H. 0

(17) What is the area of the region in the plane bounded by the curve given in polar coordinates by: $r = 4 + 2 \cos(2\theta)$.

A. 2  
B. $\sqrt{5}\pi$  
C. $4\pi$  
D. 16  
E. $18\pi$  
F. $11\pi$  
G. 32  
H. $14\pi$

(18) The function $f(x, y) = x^4 + y^4 - 4xy + 1$ has how many local minima?

A. 0  
B. 1  
C. 2  
D. 3  
E. 4  
F. 5  
G. 6  
H. 7

(19) In the sawdust mill, a log rolls down the conveyor belt and is power sanded from all sides. The log is shaped like a cylinder. When the length of the log is 10 feet and the radius is 2 feet, the length is decreasing at a rate of 3 feet/minute and the radius is decreasing at a rate of 1 feet/minute. What is the rate of change of the surface area at that time (in feet²/minute)?
(Recall that the surface area of a cylinder is $2\pi r^2 + 2\pi rl$ where $r$ is the radius and $l$ is the length of the cylinder).

A. $-10\pi$  
B. $-20\pi$  
C. $-30\pi$  
D. $-40\pi$  
E. $-50\pi$  
F. $-60\pi$  
G. $-70\pi$  
H. $-80\pi$

Answer key: BBFBGCBEGFDFCBAECDD