Final Exam - Math 114 - Fall 2014

Each problem is worth 10 points. Circle your answers. Show your work - correct answers with little or no supporting work will receive little or no credit. Partial credit may be given for wrong answers if there is significant progress towards a solution.

Name: (please print) ________________________________

Circle the name of your lecturer: Haglund Pimsner Powers Xiao

TA’s Name: (please print) ________________________________

Day of week and time of recitation: ________________________________

My signature below certifies that I have complied with Penn’s Code of Academic Integrity in completing this exam.

______________________________________________________________
Signature

The table below is for grading purposes - do not write below this line

1. _____ 8. _____  \( S_r \) _____
2. _____ 9. _____  **total** _____
3. _____ 10. _____
4. _____ 11. _____
5. _____ 12. _____
6. _____ 13. _____
7. _____ 14. _____
1. Find the values of $t$ such that the tangent lines of the curve
\[ \mathbf{r}(t) = \left( \frac{t^3}{3}, \frac{t^2}{2}, t \right) \]
at those values are parallel to the plane
\[ x + 3y + 2z = 2014. \]

(A) $t = 2$ or $3$  
(B) $t = -1$ or $-2$  
(C) $t = 0$ or $1$

(D) $t = 1$ or $3$  
(E) $t = 2$ or $5$  
(F) $t = -2$ or $2$

(G) $t = -1$ or $1$
2. Find the curvature of 
\[ \mathbf{r}(t) = (\sqrt{2}t, e^t, e^{-t}) \]
at \( t = 0 \).

(A) \( \sqrt{3} \)  \hspace{1cm} (B) \( 1/\sqrt{3} \)  \hspace{1cm} (C) \( \sqrt{5} \)  
(D) \( 1/\sqrt{5} \)  \hspace{1cm} (E) \( \sqrt{5}/8 \)  \hspace{1cm} (F) \( \sqrt{8} \)  
(G) 1
3. The tangent plane to the ellipsoid

\[ 2x^2 + 3y^2 + 10z^2 = 15 \]

at the point \((1, -1, 1)\) intersects the \(y\)-axis at the point \((0, y_0, 0)\). Find \(y_0\).

(A) 1  (B) \(-5\)  (C) 5

(D) 7  (E) 9  (F) \(-\sqrt{5}\)

(G) \(\sqrt{5}\)  (H) none of the above
4. The limit
\[
\lim_{{(x,y) \to (1,1)}} \frac{(x^2 - y^2) \cos(\pi(x^2 - y^2))}{x^4 - y^4}
\]
is:

(A) 0  
(B) 2  
(C) 1/2

(D) does not exist  
(E) \infty  
(F) 1/4

(G) 4  
(H) none of the above
5. The function \( f(x, y) = 6x^2 - 2x^3 + 6x + 3y^2 + 6y + 6xy \) has:

(A) 2 local minima

(C) 1 local maximum and 1 local minimum

(E) 1 local maximum and 1 saddle point

(G) none of the above

(B) 2 local maxima

(D) 1 local minimum and 1 saddle point

(F) 2 saddle points
6. Compute the double integral
\[
\int \int_{R} \frac{3x - y}{3x + y} \, dA,
\]
where \( R \) is the region inside the triangle with vertices \((x, y) = (0, 0), (1, 3), \) and \((2, 0)\).

(A) 3/2  (B) 5/4  (C) 1
(D) 4/5  (E) 2/3  (F) 1/2
(G) 9/2  (H) 0
7. Assume the acceleration due to gravity is 10 meters/sec$^2$ downwards. A ball is hit with a horizontal velocity of 20 meters/sec and a vertical upward velocity of 25 meters/sec. If the ball is initially 1 meter above the ground when it is hit, how high a fence will the ball clear if the fence is 80 meters away (in a horizontal direction) from where the ball is hit?

<table>
<thead>
<tr>
<th></th>
<th>Choice</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>0 meters</td>
<td></td>
</tr>
<tr>
<td>(D)</td>
<td>21 meters</td>
<td></td>
</tr>
<tr>
<td>(G)</td>
<td>√6 meters</td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td>3 meters</td>
<td></td>
</tr>
<tr>
<td>(E)</td>
<td>15 meter</td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td>47/√2 meters</td>
<td></td>
</tr>
<tr>
<td>(F)</td>
<td>19√2 meters</td>
<td></td>
</tr>
</tbody>
</table>
8. The work done by the vector field \( \langle e^z \sin(y), e^z \cos(y) + z, y + e^z \rangle \) over the path \( C \) consisting of the straight line segment

\[
(0, t, 0) \quad 0 \leq t \leq 1
\]

from \((0, 0, 0)\) to \((0, 1, 0)\), followed by the parabolic path

\[
(t^2 - 1, t, 0) \quad 1 \leq t \leq \pi
\]

from \((0, 1, 0)\) to \((\pi^2 - 1, \pi, 0)\), is

(A) \(2\pi\) \quad (B) \(\pi\) \quad (C) 0

(D) 1 \quad (E) \(1/2\) \quad (F) \(\exp(\cos(\pi^2 - 1))\)

(G) \(\sqrt{3}\)
9. Find the distance from the ellipsoid \( x^2 + y^2 + 4z^2 = 4 \) to the plane \( x + y + z - 6 = 0 \). (That is, the minimum possible distance from any point on the ellipsoid to any point on the plane). (Hint: write down a formula for the distance from a point \((x, y, z)\) on the ellipsoid to the plane \(x + y + z - 6 = 0\) and then minimize this distance).

\[
\begin{array}{ccc}
(A) \ 0 & (B) \ 1/\sqrt{6} & (C) \ 1/\sqrt{2} \\
(D) \ 2/3 & (E) \ 1 & (F) \ \sqrt{3} \\
(G) \ \sqrt{6} & & \\
\end{array}
\]
10. The value of the double integral
\[ \int_0^\pi \int_x^\pi \frac{\sin y}{y} \, dy \, dx \]
is

(A) $3\pi$  \quad (B) $\pi$  \quad (C) $\pi^2/2$
(D) 2  \quad (E) $1/2$  \quad (F) $\cos(\sqrt{\pi})$
(G) $\sqrt{2}$
11. Find the mass of the region \( R \) inside the surface \( \rho = 2 \sin \phi \) and above the \( xy \)-plane (i.e. \( z \geq 0 \)) if the density \( \delta(x, y, z) = z \), where \( \rho = \sqrt{x^2 + y^2 + z^2} \) and \( \phi \) is the angle between the positive \( z \)-axis and the ray connecting the origin to \( (x, y, z) \).

\[
\int \int \int_{R} z \, dV = \]

(A) \( \pi(2 - \sqrt{2}) \)  
(B) \( \pi \)  
(C) \( \frac{4\pi}{3} \)

(D) \( 4\pi(1 - 1/\sqrt{2})/3 \)  
(E) \( 3\pi/2 \)  
(F) \( 2\pi \)

(G) \( 2\sqrt{2}\pi \)
12. Find the volume of the region in the first octant bounded by the coordinate planes, the plane \( y = 1 - x \), and the surface \( z = \cos(\pi x / 2) \).

(A) 1/2  (B) 1/\( \pi \)  (C) 2/(3\( \pi \))
(D) 2/5  (E) 1/\( \pi^2 \)  (F) 1/3
(G) 4/\( \pi^2 \)
13. Find the volume of the region bounded above by the paraboloid $z = 9 - x^2 - y^2$, below by the $xy$-plane, and lying outside the cylinder $x^2 + y^2 = 1$.

\[
\begin{array}{llll}
(A) \ 28\pi & (B) \ 60 & (C) \ 32\pi \\
(D) \ 10\pi^2 & (E) \ 40\pi^2/3 & (F) \ 30\pi^2/4 \\
(G) \ 70
\end{array}
\]
14. Find the outward flux of $\mathbf{F}$ across the boundary of the region $D$, where

$$\mathbf{F} = (x^2, y^2, z^2)$$

and $D$ is the region cut from the solid cylinder $x^2 + y^2 \leq 1$ by the planes $z = 0$ and $z = 1$.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td>1/2</td>
<td>(C)</td>
</tr>
<tr>
<td>(D)</td>
<td>-1/2</td>
<td>(E)</td>
</tr>
<tr>
<td>(G)</td>
<td>$\pi$</td>
<td></td>
</tr>
</tbody>
</table>

| (F) | 1 |   |   |
15. Calculate \( \oint_C \mathbf{F} \cdot d\mathbf{r} \)

(the circulation of the field \( \mathbf{F} \) around the curve \( C \) in the indicated direction), where

\[ \mathbf{F} = (2y + z, x + 2z, 2x + y) \]

and \( C \) is the boundary of the triangle cut from the plane \( x + y + z = 1 \) by the first octant, counterclockwise when viewed from above.

(A) \(-3/2\)  
(B) 0  
(C) \(-1\)  
(D) 1  
(E) 2  
(F) \(-2\)  
(G) \(\sqrt{2}\)