Final Exam - Math 114 - Fall 2014

Each problem is worth 10 points. Circle your answers. Show your work - correct answers with little or no supporting work will receive little or no credit. Partial credit may be given for wrong answers if there is significant progress towards a solution.

Name: (please print) ______________________

Circle the name of your lecturer: Haglund Pimsner Powers Xiao

TA's Name: (please print) ______________________

Day of week and time of recitation:________________

My signature below certifies that I have complied with Penn's Code of Academic Integrity in completing this exam

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Signature

The table below is for grading purposes - do not write below this line

1.______	8.______	15. ______
2.______	9.______	total ______
3.______	10.______	
4.______	11.______	
5.______	12.______	
6.______	13.______	
7.______	14.______	

1. Find the values of t such that the tangent lines of the curve

$$\vec{r}(t) = \langle \frac{t^3}{3}, \frac{t^2}{2}, t \rangle$$

at those values are parallel to the plane

$$x + 3y + 2z = 2014.$$

(A) $t = 2$ or 3

(B) $t = -1$ or -2

(C) $t = 0$ or 1

(D) $t = 1$ or 3

(E) $t = 2$ or 5

(F) $t = -2$ or 2

(G) $t = -1$ or 1

2. Find the curvature of

$$\vec{r}(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$$

at $t = 0$.

(A) $\sqrt{3}$ **(B)** $1/\sqrt{3}$ **(C)** $\sqrt{5}$

(D) $1/\sqrt{5}$ **(E)** $\sqrt{8}/8$ **(F)** $\sqrt{8}$

(G) 1

3. The tangent plane to the ellipsoid

$$2x^2 + 3y^2 + 10z^2 = 15$$

at the point $(1, -1, 1)$ intersects the y-axis at the point $(0, y_0, 0)$. Find y_0.

(A) 1	**(B)** -5	**(C)** 5
(D) 7	**(E)** 9	**(F)** $-\sqrt{5}$
(G) $\sqrt{5}$	**(H)** none of the above	

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4. The limit

$$\lim_{(x,y)\to(1,1)} \frac{(x^2-y^2)\cos(\pi(x^2-y^2))}{x^4-y^4}$$

is:

(A) 0 **(B)** 2 **(C)** 1/2

(D) does not exist **(E)** ∞ **(F)** 1/4

(G) 4 **(H)** none of the above

5. The function $f(x, y) = 6x^2 - 2x^3 + 6x + 3y^2 + 6y + 6xy$ has:

(A) 2 local minima

(B) 2 local maxima

(C) 1 local maximum and 1 local minimum

(D) 1 local minimum and 1 saddle point

(E) 1 local maximum and 1 saddle point

(F) 2 saddle points

(G) none of the above

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6. Compute the double integral

$$\int\int_R \frac{3x-y}{3x+y}\, dA,$$

where R is the region inside the triangle with vertices $(x, y) = (0, 0), (1, 3),$ and $(2, 0)$.

(A) 3/2 **(B)** 5/4 **(C)** 1

(D) 4/5 **(E)** 2/3 **(F)** 1/2

(G) 9/2 **(H)** 0

7. Assume the acceleration due to gravity is 10 meters/sec^2 downwards. A ball is hit with a horizontal velocity of 20 meters/sec and a vertical upward velocity of 25 meters/sec. If the ball is initially 1 meter above the ground when it is hit, how high a fence will the ball clear if the fence is 80 meters away (in a horizontal direction) from where the ball is hit?

(A) 0 meters

(B) 3 meters

(C) $47/\sqrt{2}$ meters

(D) 21 meters

(E) 15 meter

(F) $19\sqrt{2}$ meters

(G) $\sqrt{6}$ meters

8. The work done by the vector field $\langle e^x \sin(y), e^x \cos(y) + z, y + e^z \rangle$ over the path C consisting of the straight line segment

$$\langle 0, t, 0 \rangle \quad 0 \leq t \leq 1$$

from $(0,0,0)$ to $(0,1,0)$, followed by the parabolic path

$$\langle t^2 - 1, t, 0 \rangle \quad 1 \leq t \leq \pi$$

from $(0,1,0)$ to $(\pi^2 - 1, \pi, 0)$, is

(A) 2π **(B)** π **(C)** 0

(D) 1 **(E)** $1/2$ **(F)** $\exp(\cos(\pi^2 - 1))$

(G) $\sqrt{3}$

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9. Find the distance from the ellipsoid $x^2 + y^2 + 4z^2 = 4$ to the plane $x + y + z - 6 = 0$. (That is, the minimum possible distance from any point on the ellipsoid to any point on the plane). (Hint: write down a formula for the distance from a point (x, y, z) on the ellipsoid to the plane $x + y + z - 6 = 0$ and then minimize this distance).

(A) 0	**(B)** $1/\sqrt{6}$	**(C)** $1/\sqrt{2}$
(D) $2/3$	**(E)** 1	**(F)** $\sqrt{3}$
(G) $\sqrt{6}$		

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10. The value of the double integral

$$\int_0^\pi \int_x^\pi \frac{\sin y}{y}\, dy\, dx$$

is

(A) 3π **(B)** π **(C)** $\pi^2/2$

(D) 2 **(E)** $1/2$ **(F)** $\cos(\sqrt{\pi})$

(G) $\sqrt{2}$

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11. Find the mass of the region R inside the surface $\rho = 2\sin\phi$ and above the xy-plane (i.e. $z \geq 0$) if the density $\delta(x, y, z) = z$, where $\rho = \sqrt{x^2 + y^2 + z^2}$ and ϕ is the angle between the positive z-axis and the ray connecting the origin to (x, y, z).

$$\iiint_R z\, dV =$$

(A) $\pi(2 - \sqrt{2})$ **(B)** π **(C)** $4\pi/3$

(D) $4\pi(1 - 1/\sqrt{2})/3$ **(E)** $3\pi/2$ **(F)** 2π

(G) $2\sqrt{2}\pi$

12. Find the volume of the region in the first octant bounded by the coordinate planes, the plane $y = 1 - x$, and the surface $z = \cos(\pi x/2)$.

(A) $1/2$ (B) $1/\pi$ (C) $2/(3\pi)$

(D) $2/5$ (E) $1/\pi^2$ (F) $1/3$

(G) $4/\pi^2$

13. Find the volume of the region bounded above by the paraboloid $z = 9 - x^2 - y^2$, below by the xy-plane, and lying *outside* the cylinder $x^2 + y^2 = 1$.

(A) 28π **(B)** 60 **(C)** 32π

(D) $10\pi^2$ **(E)** $40\pi^2/3$ **(F)** $30\pi^2/4$

(G) 70

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14. Find the outward flux of **F** across the boundary of the region D, where

$$\mathbf{F} = \langle x^2, y^2, z^2 \rangle$$

and D is the region cut from the solid cylinder $x^2 + y^2 \leq 1$ by the planes $z = 0$ and $z = 1$.

(A) 0

(B) 1/2

(C) 1/3

(D) −1/2

(E) −1/3

(F) 1

(G) π

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15. Calculate

$$\oint_C \mathbf{F} \cdot d\mathbf{r}$$

(the circulation of the field $\mathbf{F}$ around the curve C in the indicated direction), where

$$\mathbf{F} = \langle 2y + z, x + 2z, 2x + y \rangle$$

and C is the boundary of the triangle cut from the plane $x + y + z = 1$ by the first octant, counterclockwise when viewed from above.

(A) $-3/2$ (B) 0 (C) -1

(D) 1 (E) 2 (F) -2

(G) $\sqrt{2}$