Final Exam Math 114 Spring 2015

There are 9 problems on this exam, answer all of them. They are all multiple choice, and the first of these consists of ten true/false statements, **circle the ENTIRE phrase you deem correct among the choices given, and for the true/false statements circle the ENTIRE word "true" or "false" as the case may be**. There is partial credit given for the answers to the questions numbered 2 through 9–**you must show your work to get any credit for these**. Write answers to ALL questions on the exam paper and show your computations. There are no blue books just the exam sheets. Credit scores for each problem are indicated with the problem.

No books, tables, notes, calculators, computers (of any sort), cell-phones or any other electronic gear are allowed. One $8\frac{1}{2}'' \times 11''$ page, HANDWRITTEN (both sides OK) in your own handwriting, is allowed.

Please fill in the data below NOW.

Your name (print please)———————————————.

Your signature————————————————.

Circle Instructor's name: Haglund   Rimmer   Shatz.

Your Penn ID number————————————.

PLEASE DO NOT WRITE BELOW THIS LINE

I   VI

II   VII

III   VIII

IV   IX

V   TOTAL

FIRST QUESTION 40 TOTAL POINTS, OTHERS 20 POINTS EACH

Throughout the exam, we write f_x for $\frac{\partial f}{\partial x}$ etc.

I) True/False (4 points each answer) no partial credit :

a) Suppose two distinct lines L_1, L_2 intersect and $\vec{v}_1$, $\vec{v}_2$ are vectors along the respective lines. If Π is the plane containing the two lines and $\vec{w}$ is a vector in Π then $\vec{w} \cdot (\vec{v}_1 \times \vec{v}_2) = 0$.

    True   False

b) If $f(x,y,z)$ is a function with continuous derivatives and if $\vec{\nabla} f \,(= \vec{grad} f)$ vanishes identically, then f must be constant.

    True   False.

c) For a function, $f(x,y)$, its critical points are either maxima or minima.

    True   False.

d) Let Σ be the surface of a sphere of radius 10 with center the origin. Then, we can find a vector field, $\vec{F}$, inside and on Σ having the two properties: $div \vec{F} = 0$ and $\int \int_{\Sigma} \vec{F} \cdot \hat{n} \, d\sigma \neq 0$.

    True   False.

e) In the circle, C, of radius 1 centered at the origin, we inscribe an equilateral triangle, T. Write $\vec{w}_1, \vec{w}_2, \vec{w}_3$ for the vectors from (0,0) to the vertices of T. Then we always have $\vec{w}_1 + \vec{w}_2 + \vec{w}_3 = \vec{0}$.

    True   False.

f) If $F(x,y,z)$ is a function with continuous derivatives and (a,b,c) is a point where F vanishes, then we can always solve for z as a function of x, y near (a,b,c).

    True   False.

g) If f is a function then both $div(\vec{\nabla} f) = 0$ and $\vec{curl}(\vec{\nabla} f) = \vec{0}$.

    True   False.

h) There is no possible way of computing the area of a region by just computing some line integral around its boundary.

    True   False.

i) Suppose $\vec{r}(t)$ describes a curve in $3-$space and $\vec{r}(t) \cdot \vec{r}'(t) = 0$ for all t. Then the length of $\vec{r}(t)$ is constant.

    True   False.

j) If the position of the muzzle of a cannon and its angle of elevation is known and if we observe exactly where a shell fired from it lands, then we can determine the muzzle velocity of the cannon.

True   False.

Space for all computations and reasons below.

REMEMBER: You must show work in the next eight problems to receive any credit. A sketch of the geometry in a problem gets partial credit, also. 20 points each problem.

II) Find the arc length of the curve
$$\mathbf{r}(t) = \langle 2t, \ln t, t^2 \rangle$$
for $1 \le t \le e$.

1) $\ln 2$

2) e^4

3) $\frac{e}{2}$

4) e^2

5) 1

6) e

7) $\sqrt{e}.$

III)   Let $f(x, y) = 3xy - x^2y - xy^2$.

Let $a =$ the number of critical points the function has.

Let $b =$ the number of critical points that lead to a local maximum function.

Find a and b.

1) $a = 0, b = 0$

2) $a = 1, b = 1$

3) $a = 2, b = 1$

4) $a = 2, b = 0$

5) $a = 4, b = 1$

6) $a = 4, b = 2$

7) $a = 3, b = 2.$

IV)   The curve $C : \vec{r}(t) =< e^{t^2}, 1 + \cos(\pi t^3) >$ goes from $(1,2)$ to $(e,0)$ as t goes from 0 to 1. Let $\vec{F}$ be the vector field

$$< y + \sin(\pi x)\cos(\pi y), x + \cos(\pi x)\sin(\pi y) >.$$

What is the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$?

1) $\frac{1}{\pi}(-1 - \cos(\pi e)) - 2$

2) $\frac{1}{\pi}(2 - \cos(\pi e)) + 2$

3) $\frac{1}{\pi}(1 - \cos(\pi e)) - 2$

4) $\frac{1}{\pi}(1 + \cos(\pi e)) + 2$

5) $\frac{1}{\pi}(2 + \cos(\pi e)) - 2$

V) Suppose that the Celsius temperature at the point (x, y, z) on the sphere $x^2+y^2+z^2 = 1$ is $T = xyz^2$. What is the product of the highest and lowest temperatures on the sphere?

1) -1

2) $-\frac{1}{4}$

3) $-\frac{2}{27}$

4) $-\frac{1}{81}$

5) $-\frac{1}{64}$

6) $-\frac{1}{50}$

VI) Consider the vector field $\vec{F}(x, y, z) =$

$$< y(1 - e^z + z\cos(xyz)), z(1 + x\cos(xyz)) - xe^z, x(1 - ye^z + y\cos(xyz)) >$$

and the part of the surface, call it Σ, given by $x^2 + y^2 + z = 16$ and lying above the xy - plane. Compute the flux integral $\int\int_{\Sigma} \text{curl}\,\vec{F} \cdot \hat{n}\, d\sigma$ where $\hat{n}$ is the upward pointing unit normal on the given part of our surface.

1) 4π

2) -8π

3) 12π

4) -16π

5) 20π.

VII) Assume $|\mathbf{v}| = 2$, $|\mathbf{w}| = 3$, and the angle between $\mathbf{v}$ and $\mathbf{w}$ is $2\pi/3$. Find $|2\mathbf{v} - 3\mathbf{w}|$.

1) 11

2) 10
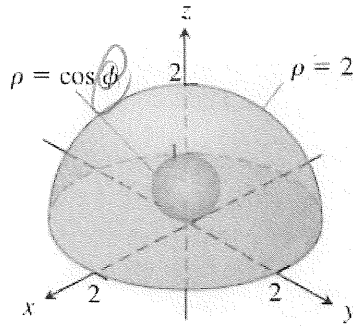
3) 9

4) 8

5) 7

6) $\sqrt{97}$

7) $\sqrt{133}$.

FIGURE 1. The solid in Problem VIII

VIII) Find the volume of the solid between the sphere $\rho = \cos\theta$ and the hemisphere $\rho = 2, z \geq 0$.

1) 15

2) $\frac{44\pi}{3}$

3) $\frac{31\pi}{6}$

4) $\frac{57\pi}{4}$

5) 5π

IX) Find the volume of the solid inside the ellipsoid $4x^2 + 9y^2 + 25z^2 = 1$.

1) $\frac{2\pi}{35}$

2) $\frac{2\pi}{45}$

3) $\frac{2\pi}{55}$

4) $\frac{2\pi}{65}$

5) $\frac{2\pi}{75}$

The next pages give further space for computations for problems II through IX.