You have 2 hours to complete this examination.

Please show all work in the space provided on your test paper and write your answers in the appropriate place below.

DO NOT DETACH THIS SHEET FROM YOUR TEST

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<th>Part I: True or False</th>
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Part III: Free Response

1. \( h(t) = \) ______________________________________________________________________

   \( \lim_{t \to \infty} h'(t) = \) ______________________________________________________________________

2. \( y(x) = \) ______________________________________________________________________

3. \( y(e) = \) ______________________________________________________________________

4. 

5. \( x = \) ______________________ \( y = \) ______________________

-----------------------------------------------Please do not write below this line-----------------------------------------------

Scores:

I. ________  II. ________  III. ________  TOTAL: ________
True/False questions

In the appropriate space on your answer sheet, label each statement as True or False. No justification is required.

1. If \( f(x, y) \) and \( f_y(x, y) \) are continuous on \( 0 < x < 3 \) and \( 0 < y < 3 \) then the initial value problem \( y' = f(x, y) \) and \( y(1) = 2 \) has a unique solution for \( 0 < x < 3 \).

Questions 2, 3, 4, 5: Consider the differential equation \( y' = g(y) \) where \( g(y) \) is given in the graph below:

![Graph](image)

2. \( y = -2, y = 1 \) and \( y = 5 \) are constant solutions of \( y' = g(y) \).

3. If the initial value \( y(0) = 4 \), the corresponding solution is increasing with a horizontal asymptote at 5.

4. If the initial value \( y(0) \) is greater than 5, the corresponding solution will be an increasing function.

5. If \( y(0) = -1 \), then the corresponding solution is increasing.

6. If \( \frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} \) for all points \((x, y)\) then \( f(x, y) = \text{constant} \).

7. If \( f(x, y, z) = x^2 + y^2 + z^2 \) then the highest point where \( f = 1 \) will have \( \nabla f = 0 \).

8. The triangle \( ABC \) with vertices \( A = 2i + 4j \), \( B = 5i - 2j \) and \( C = -3i - j \) is a right triangle.
Multiple Choice:

Work each problem in the space provided. Write the letter corresponding to your answer in the appropriate space on your answer sheet.

1. If $R$ is the region $x^2 + y^2 \leq 4$, then $\iint_R x\sqrt{x^2 + y^2} \, dA$ is equivalent to

   a) $\int_0^{2\pi} \int_0^2 r^2 \, d\theta \, dr$
   b) $\int_0^{2\pi} \int_0^2 r^3 \cos \theta \, d\theta \, dr$
   c) $4 \int_0^{2\pi} \int_0^2 r^3 \sin \theta \, d\theta \, dr$
   d) $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{x^2 + y^2} \, dy \, dx$
   e) $\int_0^{2\pi} \int_0^2 r^3 \cos \theta \, d\theta \, dr$
   f) $4 \int_0^{2\pi} \int_0^2 r^2 \, dr \, d\theta$

2. What is the area enclosed by one loop of the curve $r^2 = 2\sin \theta$.

   a) $\pi$
   b) 3
   c) 2
   d) 1
   e) 1/2
   f) 4
3. Find the length of the curve \( \mathbf{r}(t) = 2t \mathbf{i} + \sin t \mathbf{j} + \cos t \mathbf{k}, \ 0 \leq t \leq 2\pi \).
   a) \( \pi \sqrt{12} \)     b) \( \pi \sqrt{20} \)     c) \( 4\pi \)     d) \( \pi \sqrt{6} \)     e) \( \pi \sqrt{18} \)     f) \( \pi \sqrt{8} \)

4. The value of \( \alpha \) for which the vector \( \mathbf{v} = -6 \mathbf{i} + \alpha \mathbf{j} + 3 \mathbf{k} \) is parallel to the plane \( z = 2x - 5y + 7 \) is
   a) \(-3\)     b) \(-9/5\)     c) \(0\)     d) \(8/5\)     e) \(2\)     f) \(12\)
5. Let \( g(x, t, z) = \frac{1}{t}(x - z^2t) \). What is the value of \( \frac{\partial^2 g}{\partial x \partial t} \) when \( x = 1, t = 2 \) and \( z = 3 \)?

a) \(-27/3\)  
b) \(-17/2\)  
c) \(-12\)  
d) 8  
e) \(-2\)  
f) 0

6. The function \( f(x, y) = x^3 - y^2 - 3x + y + 5 \) has critical points at \( P_1 = \left( 1, \frac{1}{2} \right) \) and \( P_2 = \left( -1, \frac{1}{2} \right) \). The nature of these critical points is:

a) \( P_1 \) = maximum; \( P_2 \) = maximum  
b) \( P_1 \) = minimum; \( P_2 \) = minimum  
c) \( P_1 \) = minimum; \( P_2 \) = saddle  
d) \( P_1 \) = saddle; \( P_2 \) = maximum  
e) \( P_1 \) = saddle; \( P_2 \) = minimum  
f) \( P_1 \) = maximum; \( P_2 \) = saddle
7. Find the volume of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$.
   a) $\pi/6$  b) $\pi/4$  c) $\pi/3$  d) $\pi/2$  e) $2\pi/3$  f) $5\pi/6$

8. Use a linear approximation of the function $f(x,y) = e^{x \cos 2y}$ at $(0, 0)$ to estimate $f(0.1, -0.2)$.
   a) 1.2  b) 1.1  c) 1  d) 0.9  e) 0.3  f) 0
9. In solving the differential equation $y' + x^4 y = 0$ by use of a power series $\sum_{n=0}^{\infty} a_n x^n$, what is the first value of $n$ beyond $n = 0$ for which the coefficient $a_n$ can be non-zero?
   a) 1  b) 2  c) 3  d) 4  e) 5  f) 6

10. The initial value problem $y' = x^3(1 + e^y)$ subject to $y(0) = A$ where $A$ is a constant:
    a) always increases without bound as $x$ increases to infinity
    b) always increases to a finite limit as $x$ increases to infinity
    c) always increases to infinity at some finite $x$ value
    d) can exhibit more than one of behaviors a), b) and c), depending on the value of $A$
    e) has no solution in a neighborhood of zero
    f) has only the constant solution $y = A$
11. The general solution of the differential equation \((3x^2 + 2y^2)dx + (4xy + 6y^2)dy = 0\) is
   a) \(y - x^3 + 2x^2 + x = C\) 
   b) \(x^3 + 2x^2y^2 = C\) 
   c) \(xy^3 + 2x^2y = C\) 
   d) \(x^3 + 4xy^2 + y^2 = C\) 
   e) \(x^3 + 2xy^2 + 2y^3 = C\) 
   f) \(x^2 + xy + y^2 = C\)

12. Compute the third Picard iterate, \(y_3\), for \(y' = x + y\) subject to \(y(0) = 1\).
   a) \(y_3 = 1\) 
   b) \(y_3 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{4!}\) 
   c) \(y_3 = 1 + x^2 + \frac{x^4}{4!}\) 
   d) \(y_3 = 1 + x + x^2 + \frac{x^3}{3!}\) 
   e) \(y_3 = 1 + x + \frac{x^2}{2}\) 
   f) \(y_3 = \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\)
13. The general solution of the differential equation $y'' + 4y' + 4 y = 0$ is
   a) $y = c_1 e^{2x} + c_2 e^{2x}$  	 b) $y = c_1 e^{-2x} + c_2 e^{2x}$  	 c) $y = c_1 + c_2 x^4$
   d) $y = c_1 e^{-2x} + c_2 x e^{-2x}$  	 e) $y = c_1 e^{-x} + c_2 e^{3x}$  	 f) $y = c_1 \cos 2x + c_2 \sin 2x$

14. Solve the initial value problem $y' - \frac{1}{x} y = xe^x$ subject to $y(1) = 0$. From your solution, evaluate $y(2)$.
   a) $\frac{1}{2} e^2$  	 b) $e^2$  	 c) $2(e^2 - e)$  	 d) $e^2 \ln 2$  	 e) $e^2 + \ln 2$  	 f) $e^2 + 2$
15. Shown below are graphs of the solutions of three differential equations. The graphs are drawn using the initial condition \( y(0) = 0 \) and also \( y'(0) = 1 \) for the second order equations.

(i) \( y'' + y = 1/(10+x^2) \)
(ii) \( y'' + y = \sin(x) \)
(iii) \( y' = \cos(x + y) \)

The solution graphs shown match the differential equations (i), (ii) and (iii) in the order:

(a) 1, 2, 3  (b) 1, 3, 2  (c) 2, 1, 3  (d) 2, 3, 1  (e) 3, 1, 2  (f) 3, 2, 1
Free Response Questions:
Work each problem in the space provided. Write your answers in the appropriate spaces on your answer sheet.

1. A falling object is acted upon by gravity and air resistance; its height at time $t$ is denoted $h(t)$. There is acceleration due to gravity and air resistance, respectively $-g$ and $-k\frac{dh}{dt}$, so that the particle satisfies $h'' + kh' + g = 0$.

(a) What is the distance fallen, starting from rest, after time $t$?
(b) What is the limit as $t$ goes to infinity of $h'(t)$?

[NOTE: answers should be given in terms of $g$ and $k$.]
2. Solve $y'' - xy' + 2y = 0$ subject to $y(0) = 1$, $y'(0) = 1$ by power series expansion about $x = 0$. Write out all non-zero terms in the solution through the term in $x^5$. 
3. Solve the initial value problem \( y'' + 2y' + y = \frac{e^{-x}}{x} \) \( subject \ to \ y(1) = 0, \ y'(1) = e^{-1} \). From your solution, compute \( y(e) \).
4. Find the location of all absolute maxima of \( f(x, y) = x^3 - 3x y^2 \) on the unit disk \( \{x^2 + y^2 \leq 1\} \).
5. A business produces two products, A and B. Let $x$ and $y$ denote respectively the quantity of product A and B produced. Limitations on the company's resources require that $500x^2 + 100y$ be at most 100,000. Each unit of A produced yields a profit of $5,000 and each unit of B produced yields a $500 profit. Given the constraint, what should $x$ and $y$ be to maximize profit?