MATH 115 – Sample Final Exam 2

1. What is the probability that three randomly-selected people were born on different days of the week? (Assume that the chance of someone being born on a given day of the week is 1/7).

   (a) $\frac{1}{343}$ (b) $\frac{1}{27}$ (c) $\frac{30}{343}$ (d) $\frac{30}{49}$ (e) $\frac{35}{81}$ (f) $\frac{1}{3}$ (g) $\frac{3}{7}$ (h) $\frac{1}{720}$

2. What are the expected value μ and the variance σ^2 of a random variable X distributed on the interval $[0, 2]$ with the probability density function $f(x) = x/2$ for $0 \leq x \leq 2$?

   (a) $\mu = 4/3$, $\sigma^2 = 2$ (b) $\mu = 4/3$, $\sigma^2 = 2/9$ (c) $\mu = 4/3$, $\sigma^2 = 1/3$
   (d) $\mu = 3/2$, $\sigma^2 = 2$ (e) $\mu = 3/2$, $\sigma^2 = 2/9$
   (f) $\mu = 3/2$, $\sigma^2 = 1/3$ (g) $\mu = 1$, $\sigma^2 = 2$ (h) $\mu = 1$, $\sigma^2 = 1/3$

3. Six different pairs of socks are put in the laundry (12 socks in all, and each sock has only one mate), but only 7 socks come back. What is the expected number of pairs of socks that come back? (*Hint*: This is the sum of the expectations of the number, 0 or 1, of each individual pair that come back.)

   (a) $\dfrac{7}{22}$ (b) $\dfrac{15}{8}$ (c) $\dfrac{15}{22}$ (d) $\dfrac{30}{11}$ (e) $\dfrac{15}{4}$ (f) $\dfrac{21}{11}$ (g) $\dfrac{23}{8}$ (h) $\dfrac{51}{22}$

4. A student knows how to do 15 out of the 20 core problems for a given chapter. If the TA chooses 3 of the core problems at random for a quiz, what is the probability that the student knows how to do exactly 2 of them?

   (a) $\dfrac{27}{64}$ (b) $\dfrac{35}{228}$ (c) $\dfrac{5}{114}$ (d) $\dfrac{35}{76}$ (e) $\dfrac{55}{64}$ (f) $\dfrac{5}{38}$ (g) $\dfrac{137}{228}$ (h) $\dfrac{9}{64}$

5. 6% of Type A spark plugs are defective, 4% of Type B spark plugs are defective, and 2% of Type C spark plugs are defective. A spark plug is selected at random from a batch of spark plugs containing 50 Type A plugs, 30 Type B plugs, and 20 Type C plugs. The selected plug is found to be defective. What is the probability that the selected plug was of Type A?

   (a) $\dfrac{3}{4}$ (b) $\dfrac{10}{23}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{2}{3}$ (e) $\dfrac{15}{23}$ (f) $\dfrac{2}{23}$ (g) $\dfrac{1}{2}$ (h) $\dfrac{7}{10}$

6. The response time of communications network WEQ is an exponentially distributed random variable with mean response time of 0.1 seconds. A computer has already been waiting for 0.2 seconds for a response from WEQ. What is the probability that the computer will have to wait more than another 0.3 seconds, so that the total response time is more than 0.5 seconds?

   (a) 0.5 (b) 0.5 (c) $\dfrac{1}{\sqrt{5}}$ (d) e^{-10} (e) 0.3 (f) e^{-2} (g) e^{-3} (h) e^{-5}

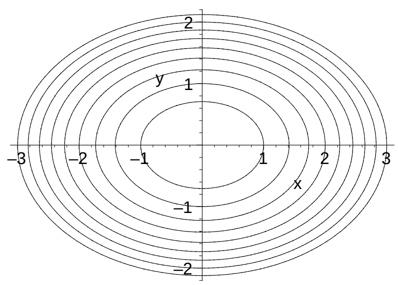
7. During business hours, the help desk for a company's computer system receives an average of 10 calls per hour. What is the probability that fewer than 3 calls come in during a randomly chosen half-hour period during business hours?

   (a) $\frac{5}{6}e^{-10}$ (b) $\frac{15}{2}e^{-10}$ (c) $\frac{23}{2}e^{-10}$ (d) $\frac{37}{2}e^{-10}$ (e) $\frac{5}{6}e^{-5}$ (f) $\frac{15}{2}e^{-5}$ (g) $\frac{23}{2}e^{-5}$ (h) $\frac{37}{2}e^{-5}$

8. A fair coin is flipped 400 times. What is the probability (to the nearest percent) that the number of heads that occurse is more than 210 and less than 240? (Use the normal approximation to the binomial distribution).

   (a) 5% (b) 12% (c) 16% (d) 29% (e) 45% (f) 55% (g) 62% (h) 79%

9. The following is a contour plot of $z = f(x, y)$.



   What is $f(x, y)$?

   (a) $x^2 + y^2$ (b) $2x^2 + y^2$ (c) $x^2 + 2y^2$ (d) $x^2 - y^2$
   (e) $x^2 - 2y^2$ (f) $2x^2 - y^2$ (g) xy (h) $x + y$

10. Suppose $f(x, y) = x^3 - 3xy + y^3 + 2$. Find the critical points of f and determine their types.

   (a) rel min at (0,0), rel min at (1,1) (b) rel min at (0,0), saddle at (1,1)
   (c) rel min at (0,0), rel max at (1,1) (d) saddle at (0,0), rel min at (1,1)
   (e) saddle at (0,0), saddle at (1,1) (f) saddle at (0,0), rel max at (1,1)
   (g) rel max at (0,0), rel min at (1,1) (h) rel max at (0,0), saddle at (1,1)

11. Let $f(x, y) = \sqrt{x^2 + 2y^2}$. Approximating $f(1.1, 1.9)$ using the total differential of f at (1,2), one gets:

   (a) 3 (b) 3.1 (c) 2.9 (d) -0.1 (e) 2.9667 (f) -0.0333 (g) 3.0333 (h) 0.0333

12. Find the point on the parabola $y = x^2$ that is closest to the point $(16, \frac{1}{2})$.

   (a) (0,0) (b) (1,1) (c) (3,9) (d) $(-1, 1)$ (e) $(-2, 4)$ (f) (2,4) (g) $(\frac{1}{2}, \frac{1}{4})$ (h) $(\frac{1}{3}, \frac{1}{9})$

13. Let T be the triangle with vertices (0,0), (1,0) and (1,2). Compute the integral $\iint_T 30x^2y\,dA$

   (a) 0 (b) 1 (c) 2 (d) 3 (e) 5 (f) 12 (g) 15 (h) 30

14. Calculate the volume of the solid bounded above by the graph of $z = e^{-x^3/2}$ over the region R in the xy-plane bounded by the graphs of $y = x^2$, $y = 2x^2$ and by the vertical lines $x = 0$ and $x = 1$.

(a) 1 (b) e (c) $\frac{2}{3}(1 - e^{-1/2})$ (d) $\frac{2}{3}(1 + e^{-1/3})$
(e) $\frac{3}{2}(1 - e^{-2/3})$ (f) $\frac{1}{3}(1 + e^{1/2})$ (g) $\frac{3}{2}(1 + e^{-2/3})$ (h) $\frac{1}{3}(1 - e^{-1/3})$

15. The sum of the entries of the inverse of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 1 (d) -1 (e) 0 (f) 2 (g) $-\frac{1}{3}$ (h) 3

16. A simple board game has four fields A, B, C, and D. Once you end up on field A you have won and once you end up on field B you have lost. From fields C and D you move to other fields by flipping a coin. If you are on field C and you throw a head, then you move to field A, otherwise to field D. From field D, you move to field C if you throw a head, and otherwise you mover to field B.

Suppose that you start in field D. What is the probability that you will win (i.e., what is the probability that you will end up on field A)? (*Hint*: to compute this you need to compute the stable matrix of the appropriate absorbing stochastic matrix.)

(a) 0 (b) 1 (c) 0.25 (d) 0.5 (e) 0.75 (f) $\frac{1}{6}$ (g) $\frac{1}{3}$ (h) $\frac{2}{3}$

17. Find *all* of the solutions of the following system of linear equations:

$$\begin{aligned} x + 2y + 4z + w &= 1 \\ y + 2z + w &= 1 \\ w &= -1 \end{aligned}$$

(a) $x = -2,\ y = 2,\ z = 1,\ w = -1$ (b) $x = -2,\ y = 2 - 2t,\ z = t,\ w = -1$
(c) $x = -2,\ y = -2,\ z = 2,\ w = -1$ (d) $x = -2t,\ y = -2,\ z = t,\ w = -1$
(e) $x = -2,\ y = 0,\ z = 1,\ w = -1$ (f) $x = 2,\ y = 2t - 2,\ z = t,\ w = -1$
(g) $x = 2 + t,\ y = 2t,\ z = t,\ w = -1$ (h) $x = 2 + 2t,\ y = t - 2,\ z = t,\ w = -1$

18. For what value of k does this system of equations have at least one solution?

$$\begin{aligned} x - y + z &= 2 \\ x + y - z &= 4 \\ x + 3y - 3z &= k \end{aligned}$$

(a) 0 (b) 1 (c) -1 (d) 3 (e) $\frac{3}{2}$ (f) 4 (g) 6 (h) no value

19. Suppose A is a 2×2 matrix and $A^2 = AA = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 1 & 7 \\ 0 & 8 \end{bmatrix}$. What is the entry in the second row, second column of A?

(a) $A_{22} = 0$ (b) $A_{22} = 1$ (c) $A_{22} = -1$ (d) $A_{22} = 3$
(e) $A_{22} = -2$ (f) $A_{22} = 2$ (g) $A_{22} = \frac{1}{2}$ (h) $A_{22} = \frac{1}{3}$

20. The maximum value of $P = 3x + 4y$ subject to $2x + y \leq 5$, $x + y \leq 4$ and $x, y \geq 0$ is attained at

(a) (0,5) (b) (0,4) (c) (0,0) (d) (1,3) (e) (2.5,0) (f) (4,0) (g) (2,1) (h) (2,2)