

## Math 115 Calculus, Part II with Probability and Matrices.

Functions of several variables, partial derivatives, multiple integrals, differential equations; introduction to linear algebra and matrices with applications to linear programming and Markov processes. Elements of probability and statistics. Applications to social and biological sciences. Use of symbolic manipulation and graphics software in calculus.

Prerequisite(s): Math 104. This is a Formal Reasoning course.

Math 115 can now be used both as pre-req for Stat 430 and it can be used for the Stat concentration.

**Texts:** [C] *Thomas' Calculus Early Transcendentals Second Custom Edition for the University of Pennsylvania* Pearson 2014. Package ISBN : 978-1-269-95070-1

[P] *Schaum's Outlines Probability*, 2nd edition, by Seymour Lipschutz, McGraw Hill ISBN 10: 0-071-75561-6 ISBN 13: 978-0-071-75561-0

[H] *Linear Algebra* by Jim Hefferon, St. Michael's College; text downloadable for free from <http://joshua.smcvt.edu/linearalgebra/book.pdf>

Section	Title	Core Problems
[C] Thomas' Calculus Early Transcendentals Second Custom Edition for the University of Pennsylvania		
12.1-12.5	Review of Vectors, Dot Products, Lines and Planes in Space	12.2: 17, 19, 25 12.3: 13, 25 12.5: 1, 5, 8, 26
14.1	Functions of Several Variables	3, 9, 14, 18, 31, 32, 33, 34, 35, 36, 39, 50, 55, 62, 65
14.2	Limits and Continuity in Higher Dimensions	1, 9, 16, 27, 32, 41, 49, 56, 61
14.3	Partial Derivatives	5, 22, 26, 39, 46, 54, 63, 65, 73, 83, 90
14.4	The Chain Rule	3, 7, 12, 14, 25, 31, 35, 41, 45, 50, 51
14.5	Directional Derivatives and Gradient Vectors	3, 8, 13, 21, 26, 29, 34, 39
14.6	Tangent Planes and Differentials	3, 9, 15, 19, 24, 29, 33, 42, 47, 49, 54, 58
14.7	Extreme Values and Saddle Points	2, 17, 31, 41, 44, 49, 59, 65
14.8	Lagrange Multipliers	5, 11, 20, 29, 31, 42, 43
15.1	Double and Iterated Integrals over Rectangles	1, 14, 19, 22, 27
15.2	Double Integrals over General Regions	1, 9, 19, 26, 35, 51, 57, 67, 71, 73, 78, 84

### [P] Schaum's Outline Probability

2.1-2.7	Sets and Elements, Subsets, Set Operations, Finite and Countable Sets, Counting Elements in Finite Sets	2.39, 2.40, 2.41, 2.45-2.49, 2.51, 2.52, 2.54, 2.57, 2.59-2.61, 2.64-2.67, 2.75, 2.76
3.1, 3.2, 3.4, 3.5	Sample Space and Events, Finite Probability Spaces, Infinite Sample Spaces	3.41, 3.42, 3.44-3.48, 3.50-3.52, 3.54, 3.59, 3.60
4.1-4.6	Conditional Probability, Finite Stochastic and Tree Diagrams, Partitions, Total Probability and Bayes' Formula	4.41-4.48, 4.53-4.60, 4.63-4.66, 4.69, 4.70, 4.72, 4.74, 4.75, 4.81, 4.84
5.1, 5.2, 5.5	Random Variables and Expected Value, Mean, Variance, and Standard Deviation	5.54-5.57, 5.59, 5.60, 5.69, 5.73
6.1-6.3	Binomial Distribution, Expected Value and Standard Deviation	6.52, 6.56, 6.57, 6.62, 6.63, 6.84, 6.89
7.1-7.6	Vectors and Matrices, Probability Vectors	7.23, 7.24, 7.26-7.30, 7.32, 7.35, 7.36,

	and Stochastic Matrices, Transition Matrix of a Markov Process, State Distributions, Regular Markov Processes and Stationary, State Distributions	7.38, 7.39
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[H] Linear Algebra by Jim Hefferon

I.1	Solving Linear Systems: Gauss' Method	1.17, 1.18 a-d, 1.19, 1.20, 1.22 a, 1.25
I.2	Solving Linear Systems: Describing the Solutions Set	2.15, 2.17, 2.18 a-d, 2.19 b, 2.20, 2.25
I.3	Solving Linear Systems: General = Particular + Homogeneous	3.15 a-c, 3.17, 3.21
II.1	Vectors in Space	1.1, 1.2, 1.3, 1.4, 1.7
II.2	Length and Angle Measurements	2.12, 2.13, 2.14, 2.18, 2.20
III.1	Gauss-Jordan Reduction	1.7 a-c, 1.8 a and b, 1.9 b and c
III.2	Row Equivalence	2.10 a-c, 2.23

SAMPLE EXAM QUESTIONS (available from the Math Dept's Math 115 Web Page: <http://www.math.upenn.edu/ugrad/calc/m115/>) also form a part of the core.

The core problems indicate the kind of basic problems you will need to be able to solve by hand. They also provide a guide to the basic level of difficulty to be expected on the final exam.

Additional Problems on Bivariate Probability Distributions:

1. Suppose the joint probability density function of a pair of random variables is given by

$$f(x, y) = \begin{cases} \frac{1}{2} & \text{if } 0 < x < 2 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} . \text{ Compute } \text{Prob}(X > Y).$$

2. Suppose the joint probability density function of a pair of random variables is given by

$$f(x, y) = \begin{cases} xy & \text{if } 0 < x < 2 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases} . \text{ Compute } \text{Prob}(X > Y).$$

3. Suppose the joint probability density function of a pair of random variables is given by

$$f(x, y) = \begin{cases} 6e^{-(2x+3y)} & \text{if } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases} . \text{ Assume } a > 0.$$

Compute  $\text{Prob}(X > a)$ ,  $\text{Prob}(Y > a)$ ,  $\text{Prob}(\min(X, Y) > a)$ , and  $\text{Prob}(X > Y)$ .

4. Suppose the joint probability density function of  $n$  random variables  $X_k$ ,  $k = 1, 2, \dots, n$  is

$$\text{given by } f(x) = \begin{cases} 2^{-n} & \text{if } 0 < x_k < 2, k = 1, 2, \dots, n \\ 0 & \text{otherwise} \end{cases} . \text{ Let } Y = X_1 + X_2 + \dots + X_n .$$

Compute  $E(Y)$  and  $E(Y^2)$ . Hint. For independent random variables we have the means and variances add.

Note: All sections of Math 115 have a COMMON FINAL EXAM