This Exam is being taken as (check one): ☐ Makeup Final ☐ AP Exam

Professor (makeup, check one): ☐ Prof. Crotty ☐ Prof. Schneiderman ☐ Prof. Vogel

- There are 10 questions on this test. You are required to do all 10. Each question is worth 10 points.
- Show how you obtained your answers—problems showing only answers will be given only partial credit and then only if the results are completely correct.
- When statements or explanations are required, be brief and to the point.
- You have a maximum of two hours for this examination.

Please show all work in the space provided on your test paper and write your answers in the appropriate place on each page. If you need more space, use the back of the page facing the problem on which you are working.

DO NOT DETACH THIS SHEET FROM YOUR TEST

Scores:

1. (10) 6. (10) 2. (10) 7. (10) 3. (10) 8. (10) 4. (10) 9. (10) 5. (10) 10. (10)
1. Solve the following system of linear equations by the method of your choice:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x_1 - 3x_2 + x_3 = -9$</td>
<td>$x_1 =$ ________</td>
</tr>
<tr>
<td>$-x_1 + 2x_2 - 4x_3 = -3$</td>
<td>$x_2 =$ ________</td>
</tr>
<tr>
<td>$4x_1 - x_2 + 5x_3 = -9$</td>
<td>$x_3 =$ ________</td>
</tr>
</tbody>
</table>
2. Diagonalize the following matrix, if possible, or explain why the matrix is not diagonalizable; if the matrix can be diagonalized, find \( P \), the diagonalizing matrix, its inverse, \( P^{-1} \), and \( D \), the resulting diagonal matrix:

\[
A = \begin{bmatrix}
1 & 0 & 0 \\
1 & 2 & 0 \\
-3 & 5 & 2
\end{bmatrix}
\]

Answers here:

\[
P = \begin{bmatrix}
\_ & \_ & \_ \\
\_ & \_ & \_ \\
\_ & \_ & \_ \\
\end{bmatrix} \quad P^{-1} = \begin{bmatrix}
\_ & \_ & \_ \\
\_ & \_ & \_ \\
\_ & \_ & \_ \\
\end{bmatrix} \quad D = \begin{bmatrix}
\_ & \_ & \_ \\
\_ & \_ & \_ \\
\_ & \_ & \_ \\
\end{bmatrix}
\]

or if \( A \) cannot be diagonalized, explain why here:

__________________________________________________________________________

Show work here:
3. Compute $\text{curl } \mathbf{F}$ and $\text{div } \mathbf{F}$ if $\mathbf{F} = (-x + y)\mathbf{i} + (y + z)\mathbf{j} + (x - z)\mathbf{k}$.

Answers here:

$$\text{curl } \mathbf{F} = \quad \text{div } \mathbf{F} =$$

Show work here:
4. a) Show that \( \mathbf{F}(x, y, z) = (2xz + \sin y)i + (x \cos y)j + x^2k \) is a conservative vector field.

b) Use the result of part a) to evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) along the curve

\[ C : \mathbf{r}(t) = (\cos t)i + (\sin t)j + tk, \quad 0 \leq t \leq 2\pi \]  

[\text{S3:14-3-17}]

Answers here:

\[
\begin{array}{l}
\text{a) } \mathbf{F} \text{ is conservative because:} \\
\text{b) } \int_C \mathbf{F} \cdot d\mathbf{r} =
\end{array}
\]

Show work here:
5. Solve the system of differential equations \( \frac{dx}{dt} = x + 2y \), \( \frac{dy}{dt} = 4x + 3y \). You may express your solution in any suitable form. [zc9.4.1.1]

Answer here:

Solution:

Show work here:
6. Use Green's Theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = (-16y + \sin x^2)i + (4ey + 3x^2)j$ and $C$ is as shown in the graph below.

Answer here: $\int_C \mathbf{F} \cdot d\mathbf{r} =$

Show work here:

$C: C_1 \cup C_2 \cup C_3$

$C_2: x^2 + y^2 = 1$

$C_3: y = -x$

$C_1: y = x$
7. Use the Divergence Theorem to evaluate \( \iiint_S (F \cdot n) \, dS \) if \( F \) is given by \( F = xyi + y^2j + z^3k \) and \( S \) is the unit cube defined by \( 0 \leq x \leq 1, \, 0 \leq y \leq 1, \, 0 \leq z \leq 1 \).

Answer here:
\[ \iiint_S (F \cdot n) \, dS = \]

Show work here:
8. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = (x + 2z)i + (3x + y)j + (2y - z)k$ and $C$ is the curve of intersection of the plane $x + 2y + z = 4$ with the coordinate planes (see graph below).

Answer here:

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \quad \quad \quad \quad$$

Show work here:
9. Find two power series solutions of $y'' + x^2 y = 0$ about $x = 0$. Write out the first three non-zero terms of each series.

Answers here:

<table>
<thead>
<tr>
<th>$y_1 =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_2 =$</td>
</tr>
</tbody>
</table>
10. a) Consider the differential equation \((x^3 + 4x)y'' - 2xy' + 6y = 0\). For which, if any of its singular points does Frobenius' Theorem guarantee at least one solution? How do you know?

b) Assume you find power series solutions of the equation in part (a) about the ordinary point \(x = 4\). What is the "guaranteed" radius of convergence of these solutions? How do you know?

Answers here:

a) ____________________________________________________________

b) ____________________________________________________________

Show work here: