Solutions Final Exam
Math 240 – Spring 2014

I. a) F; b) F; c) T; d) T; e) T; f) T; g) T; h) F; i) F; j) F

II. 4)

III.

$$e^{tA} = e^t \begin{pmatrix} 1+2t & -t \\ 4t & 1-2t \end{pmatrix}.$$

IV.

$$x(t) = -\frac{1}{2} + c_1 e^{2t} + c_2 e^{-t}$$

V. 2)

$$y(t) = -\frac{1}{2}\sin(2t) + \cos(2t)$$

VI. a) 2; b) 3; c) 2

VII. a) For all λ in $\mathbb{R}$ and $p(X)$ in S, we have that $\lambda p(X)$ is in S. Also, for all a_1, b_1, a_2, b_2 in $\mathbb{R}$, by letting $p_1(X) = a_1 x^3 + b_1 X$ and $p_2(X) = a_2 x^3 + b_2 X$, we have that $p_1(X) + p_2(X) = (a_1 + a_2)x^3 + (b_1 + b_2)X$, which belongs to S. Thus, S is a subspace.
b) Basis in S: $B_1 = \{X, x^3\}$. Basis in $\mathbb{R}^2$: $B_2 = \{(1,0),(0,1)\}$. Then

$$[L]_{B_1}^{B_2} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

Because the matrix $[L]_{B_1}^{B_2}$ is invertible, the linear transformation L is also invertible.

VIII. $y(x) = x + 1 - e^x \sin x$

IX. $y(x) = e^x u(x)$, where $u(x)$ solves $xu''(x) + u'(x) = 0$, that is, $u(x) = c_1 \ln x + c_2$

X.

$$x(t) = c_1 e^t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^{-3t} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_3 e^{-3t} \begin{pmatrix} 0 \\ t \\ 1 \end{pmatrix}.$$

XI.

$$\begin{pmatrix} -1 & 2 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & -1 \end{pmatrix}$$

XII. 3)