1. Find all values of the cube root of $-4\sqrt{2} + i4\sqrt{2}$. Draw a picture of the roots in the complex plane.

2. Find all values of arcsin 2 (or $\sin^{-1} 2$). Draw a picture of these values in the complex plane.
3. Let $f(z) = u(x,y) + iv(x,y)$ be a function of a complex variable $z = x + iy$. For which of the functions $u(x,y)$ below does there exist a (real-valued) function $v(x,y)$ so that $f(z)$ is analytic? If it exists, find $v(x,y)$, and calculate the complex derivative $f'(z)$.

a) $u(x,y) = x^2 + 2y$

b) $u(x,y) = x^2 + 2x - y^2$

c) $u(x,y) = e^{-x}(x \sin y - y \cos y)$
4. Evaluate the integral

\[ \int_C z \overline{z} \, dz, \]

where \( C \) is the segment of the parabola \( y = x^2 \) between the points \((0, 0)\) and \((1, 1)\) (i.e., from \( z = 0 \) to \( z = 1 + i \)). The curve \( C \) is oriented from left to right.
5. Evaluate the integral
\[ \int_C \frac{1}{z^4} \, dz. \]

The path of integration \( C \) is the semicircle \(|z| = 1\) in the lower half plane, which starts at \( z = 1 \) and ends at \( z = -1 \).
6. For each of the functions \( f \) below, give the first four terms of the Laurent series centered at the indicated singularity \( z_0 \).

a) \( f(z) = z^2 \sin \frac{1}{z} \) at \( z_0 = 0. \)

b) \( f(z) = \frac{1}{z^3 + 2z} \) at \( z_0 = 0. \)

c) \( f(z) = \frac{\cos z}{z - \pi/2} \) at \( z_0 = \pi/2. \)

Then answer the following questions in each case: What is the residue of \( f \) at \( z_0 \)? Is the singularity at \( z_0 \) removable, essential, or a pole? If it is a pole, what is its order?
7. Evaluate the contour integral

\[ \oint_C \frac{\cos z/4}{z^2 - \pi z} \, dz \]

for each of the following (positively oriented) circles \(c\):

a) \( |z| = 4 \), b) \( |z - 4| < 2 \), c) \( |z - 2| = 1 \).
8. Evaluate the contour integral

\[ \oint_C \frac{z^2}{\cos z} \, dz. \]

The contour \( c \) is composed of four straight line segments \( c_1, c_2, c_3, c_4 \) (in this order), where
\( c_1 \) connects \((2, -2)\) to \((2, 2)\), \( c_2 \) connects \((2, 2)\) to \((-2, -2)\), \( c_3 \) connects \((-2, -2)\) to \((-2, 2)\), and \( c_4 \) connects \((-2, 2)\) to \((2, -2)\), closing the loop.
9. Evaluate the real integral
\[ \int_{-\infty}^{+\infty} \frac{x^2 + 1}{(x^2 + 2)^2} \, dx. \]
10. Evaluate the real integral

$$\int_0^{2\pi} \frac{1}{a + \cos^2 \theta} \, d\theta,$$

where $a = -\frac{1}{2} + \frac{1}{2}\sqrt{5}$ (incidentally: $a$ is the value of the golden ratio).

**Bonus (5 extra points):** Find an expression for the value of the integral for general $a > 0$. 