MATH 360 Homework 10

Due 12 April 2013

1. (Marsden-Hoffman's problem 4.42) For $x > 0$, define $L(x) = \int_1^x \frac{1}{t} dt$. Show the following *using only this definition of L and theorems about the definite integral.*
   i. $L(x)$ is a strictly increasing function.
   ii. $L(xy) = L(x) + L(y)$.
   iii. $L(x)$ grows without bound as x grows without bound and decreases without bound as $x \to 0$. (*Hint.* Use (ii).)
   iv. $L'(x) = \frac{1}{x}$
   v. $L(1) = 0$
   vi. If $g(x)$ is a function with $g(x) = g'(x)$, $g(x) > 0$, and $g(0) = 1$, then $L(g(x)) = x$ for all $x \in \mathbb{R}$.
   vii. If $g(x)$ is a function with $g(x) = g'(x)$, $g(x) > 0$, and $g(0) = 1$, then $g(L(x)) = x$ for all $x > 0$.
2. For each of the following double-sequences, compute $\lim_n \lim_m s_{m,n}$, $\lim_m \lim_n s_{m,n}$, and $\lim_n s_{n,n}$:
   i. $s_{m,n} = \frac{m}{n}$
   ii. $s_{m,n} = \frac{m-n}{m+n}$
   iii. $s_{m,n} = \left(\frac{1}{m}\right)^n$
   iv. $s_{m,n} = \sin\left(\frac{m}{n}\right)$
3. (Rudin's 7.1) Let $\mathcal{F}$ be a family of real-valued functions from some metric space (M, d): $M \to \mathbb{R}$. We say the family $\mathcal{F}$ is *uniformly bounded* if there is some $B \in \mathbb{R}$ so that for any $f \in \mathcal{F}$ and any $x \in M$, $|f(x)| \leq B$. Suppose that $(f_n)_{n\in\mathbb{N}}$ is a uniformly convergent sequence of functions, each of which is bounded. Show that the sequence is uniformly bounded. (*Hint.* The limit of the uniformly convergent sequence is bounded.)