MATH 600: Geometric Analysis and Topology I

Instructor. Andrew A. Cooper, `ancoop@math.upenn.edu`

Office Hours. DRL 3N4A, TW 2-3pm or by email appointment

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Goal. This course is intended as an introduction to differential geometry for first-year graduate students. Its goal is to prepare you for a preliminary examination in differential geometry.

Content. This course is an introduction to the basic object of most differential geometry, the *smooth manifold*. We will cover the following core topics:

smooth manifolds, smooth structures, smooth maps and differentials, bump functions, vector fields, the tangent bundle, general vector bundles, tensors, differential forms and exterior calculus, submanifolds, implicit function theorems, integration

Time permitting, we will cover some other topics:

Sard's theorem, transversality, de Rham cohomology, Whitney's embedding theorem, volumes, Poincaré-Hopf theorem, Morse theory

Prerequisites. The formal prerequisites are point-set topology (e.g. MATH 500) and differential geometry of curves and surfaces (e.g. MATH 501).

In addition, a great deal of what we will discuss comes down to linear algebra. If you have not had a formal theoretical linear algebra course, expect to learn a lot!

Texts.

Lee: The "required" text is John Lee's "Introduction to Smooth Manifolds". This book will serve as the backbone of the course, and will be the source of many of the homework exercises. *Lee has been declared "out of print" pending the release of the second edition in October 2012. However, new copies still appear to be in stock via Amazon and other booksellers, and there is no shortage of used copies. Please let me know if you have difficulty obtaining a copy.*

Milnor: The "recommended" text is John Milnor's "Topology from the Differentiable Viewpoint". This is a jewel of a book that anyone working in geometry or topology absolutely must have. I will take a few topics from it. *Milnor is available from the bookstore, but may also be obtained via Amazon or other sources, which may be cheaper.*

other books: Other books that may be helpful include: F. Warner, "Foundations of Differentiable Manifolds and Lie Groups"; M. Spivak, "Comprehensive Introduction to Differential Geometry" (vol. I); Guillemin and Pollack, "Differential Topology". For Spivak, I recommend the Third Edition (a quick comparison at the First Edition should convince you that LaTeX is among one of the more important human achievements of the latter half of the 20th century).

references for prequisite material: M. Spivak, "Calculus on Manifolds"; J. Munkres, "Topology"; M. do Carmo, "Differential Geometry of Curves and Surfaces".

Websites such as Wikipedia and WolframAlpha can also be quite helpful in getting a first-pass understanding of certain topics, though they should be treated with a bit more caution. There is a lot of wrong information out there.

Evaluation. Your grade will be determined by performance on the following items:

Homework: There will be approximately biweekly homework assignments. Unless otherwise stated, homework is due on Friday afternoon at 4.30pm in my mailbox in the Mathematics Department office.

Homework should be written neatly and carefully. If you can, please type your homework using LaTeX–this is a skill that is absolutely essential for mathematics graduate students, and helpful for anyone else besides.

Homework will account for 60% of the course grade.

Exams: There will be two exams. The first will be an in-class exam which will test your ability to recognise examples of various objects we have discussed. The second will be a take-home exam which you will have one week to complete and will require creative application of the theorems we have learned. Each exam will account for 20% of the course grade.

Collaboration: To get the most out of the course, you should attempt the homework alone, but you are free to consult other students, Ms. Kjuchukova, or me, in preparing your homework.

In your homework, please cite all sources you consulted–including the names of fellow students.

For take-home exams, you may consult Lee and Milnor as well as your class notes. You may not use any other sources–including your classmates, other students, books, internet sources, etc.

Seminar. There will be a weekly seminar chaired by Ms. Kjuchukova (and not attended by me). Participants will present fifty-minute talks on topics related to the course content. In addition to presenting in the seminar, you should be an active audience member. Participation is required for graduate students. Participation is optional, but very useful, and may incur extra credit for undergraduates. If you would like to present a particular topic, that is fine; if you do not have a topic in mind either Ms. Kjuchukova or I can suggest a topic for you.