Math 114 HW 2

due Tuesday, 5/31

1. (Spring 2013) Find the equation of the plane that passes through (1, 3, 2) and contains the line

\[ \begin{align*}
  x &= 1 + t \\
  y &= -1 - 2t \\
  z &= 3 + 2t 
\end{align*} \]

What is the y-coordinate of the point where this plane intersects the y-axis?

2. (Spring 2013) Let \( \mathbf{r}(t) = \sqrt{2} \cos t \mathbf{i} + \sqrt{2} \sin t \mathbf{j} + t \mathbf{k} \). Using the parametric equations for the line tangent to the curve at \( t = \frac{\pi}{4} \), find the coordinates of the point where the tangent line intersects the xy-plane.

3. (Fall 2011) Find \( \mathbf{r}(t) \) if

\[ \frac{d^2 \mathbf{r}}{dt^2} = \langle t^2, 1, -t \rangle \]

\[ \frac{d\mathbf{r}}{dt}(1) = \langle \frac{2}{3}, 0, -\frac{1}{2} \rangle \]

\[ \mathbf{r}(0) = \langle 1, -1, 0 \rangle \]

What is the value of \( \mathbf{r}(1) \)?

4. (Spring 2012) A particle in space accelerates according to \( \mathbf{a}(t) = 2 \mathbf{i} + (t^2 - 1) \mathbf{j} + \mathbf{k} \) with initial velocity \( \mathbf{v}_0 = 3 \mathbf{i} + 4 \mathbf{j} \) and initial position \( \mathbf{r}_0 = \mathbf{i} + 5 \mathbf{k} \). What is the position of the particle at time \( t = 2 \)?

5. (Fall 2013) A projectile is launched from the ground at an angle of \( \frac{\pi}{4} \), and with an initial speed of \( 48 \sqrt{2} \text{ feet/sec} \). How many seconds does it take a projectile to reach a height of 32 feet for the first time? Take gravitational acceleration \( g \) to be \( 32 \text{ feet/sec}^2 \).

6. (Spring 2013) Assume the acceleration of gravity is \( 10 \text{ m/sec}^2 \) downwards. A cannon ball is fired at ground level. If the cannon ball rises to a height of 80 meters and travels a distance of 240 meters before it hits the ground, what is the magnitude of the initial velocity in meters per second?

7. (Fall 2010) Let \( \mathbf{r}(t) = \langle 2t, t^2, \ln t \rangle \). Find the arclength for \( 1 \leq t \leq e \).

8. (Fall 2013) A curve \( C \) in 3-space is defined by

\[ \mathbf{r}(t) = (4 \cos t) \mathbf{i} + (4 \sin t) \mathbf{j} + 3t \mathbf{k} \]

Find the point \( p_0 \) on the curve \( C \) which has distance \( \frac{5\pi}{4} \) from the point \( (4, 0, 0) \), as measured along the curve.
9. (Fall 2009) A kid is riding a roller coaster in an amusement park. Part of the track follows the curve

$$\vec{r}(t) = \left< t, t^2, \frac{2}{3} t^3 \right>, \quad 0 \leq t \leq 2$$

How long is this part of the coaster track?

10. (Spring 2005) Find the unit tangent vector to the curve

$$\vec{r}(t) = e^{2t} \cos t \hat{i} + e^{2t} \sin t \hat{j} + e^{2t} \hat{k}$$

at the point where \( t = \frac{\pi}{2} \).