(prop) The dgsm cat. $C_*(\overline{N}_{open}, \det^d)$ is quasi-isomorphic to

$$\mathrm{ob}\,\mathcal{OC}^d - C_*(\overline{N}_{open}, \det^d) \qquad C_*(\tilde{N}, \det^d)$$

$$\mathrm{ob}\,\mathcal{OC}^d - \mathcal{O}^d \longleftarrow \mathcal{OC}^d$$

✲ $\mathcal{D}^d(\alpha, \beta) = C_*^{cell}(G(\alpha, \beta)\, \det^d) \otimes k$.

∘ explain ; element ; RS → chain complex

$\langle \mathcal{D}^d_{open} \underset{q.i}{\sim} \mathcal{O}^d$ / ∃ q. equivalence between $\left\{ \begin{array}{l} \mathrm{ob}\,\mathcal{OC}^d - \mathcal{D}^d_{open} - \text{bimod} \\ \mathrm{ob}\,\mathcal{OC}^d - \mathcal{O}^d - \text{bimod} \end{array} \right\}$

$$\begin{array}{c} \mathcal{D}^d \\ \updownarrow \\ \mathcal{OC}^d \end{array}$$

(step2) Generator. & relation description.

① $\underline{\mathcal{O}^d_\wedge \sim \mathcal{D}^d_{open}}$ ② $\underline{\mathcal{OC}^d_\wedge \sim \mathcal{D}^d}$ as SymOb $- \mathcal{O}^d_q$ module

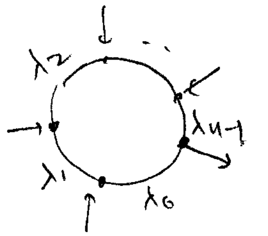
∵ (Reminder) Representative of morphism.

$\mathcal{D}^d_{open}$; no-annulus (except exceptional)

$\cup$ Disks, ≤ 2 . . . .

$\mathcal{D}^+_{open}$; disj union of disks / connected component

having precisely one outgoing marked point



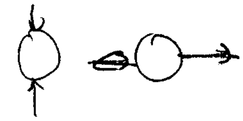
$D^+(\lambda_0, \ldots, \lambda_{n-1}) \in \mathrm{Hom}(\{\lambda_0, \ldots, \lambda_{n-1}\}, \{\lambda_0, \lambda_1\})$ $n \geq 1$

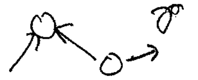
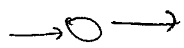
(claim 1) $\mathcal{D}^+_{open}$ is finitely gen'd. as sym-mod cat / ob $\mathcal{D}^d_{open}$ by those

① $D^+(\lambda_0, \ldots, \lambda_i, \lambda_i, \ldots, \lambda_{n-1}) \circ D^+(\lambda_i) = 0$

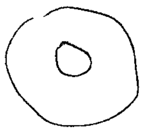
② $D^+(\lambda_0, \lambda_0, \lambda_1) \circ D^+(\lambda_0) = 0$

③ $D^+(\lambda_0, \lambda_1, \lambda_1) \circ D^+(\lambda_1) = 0$.

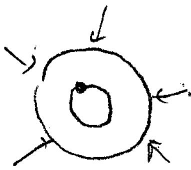
(claim 2) $\mathcal{D}^d_{open}$ is freely gen'd by $\mathcal{D}^+_{open}$ & .

(relation :  $\Rightarrow$ 

• $D(\lambda_0, \cdots, \lambda_{n-1})$ is cyclically symmetrically.)

(ex)  same thing //

Next [2] GR for $\mathcal{D}^d$.

$\lambda_0 \cdots, \lambda_{n-1}$ ordered set of D-branes 

$A(\lambda_0, \cdots \lambda_{n-1}) \in \mathcal{D}^d(\{\lambda_0, \cdots, \lambda_{n-1}\}^c, (1,0))$

Note that $\underbrace{(n-1) \text{ dim'n}}_{\text{why?}}$ in $\mathcal{D}^d$

(Thm. 6.2.4). $Ob\mathcal{OC} - \mathcal{D}^d_{open}$ bimodule $\mathcal{D}^d$ is freely gen'd. by $A(\lambda_0, \cdots \lambda_{n-1})$, $1 \in \mathcal{D}^d_{open}(\alpha, \alpha) \subset \mathcal{D}^d(\alpha, \alpha)$.

• If we glue the disc w/ one boundary to any of the open marked points of $A(\lambda_0, \cdots, \lambda_{n-1})$. except lying bet λ_{n-1} and λ_0, we get 0.

• $Id_\alpha \amalg Id_\beta \cong Id_{\alpha \cup \beta}$

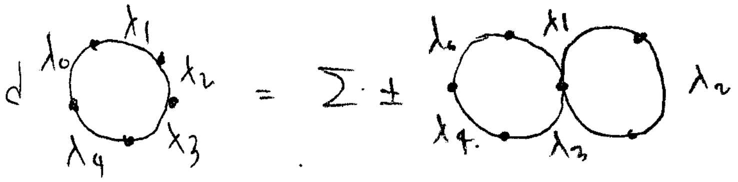
As $Ob\mathcal{OC} - \mathcal{D}^d_{open}$ bimodule ; $\underline{\underline{\mathcal{D}^d}}$

$(A, B) \longmapsto \mathcal{D}_{open}(B, A)$

if we take $A = \alpha = 0$. disc w/ all incoming boundary pt

(Def). $\mathcal{D}^+$ be $Ob\mathcal{OC} - \mathcal{D}^+_{open}$ bimod w/ same generators and relations as $\mathcal{D}^d$

$$\mathcal{D}^d = \mathcal{D}^+ \otimes_{\mathcal{D}^+_{open}} \mathcal{D}^d_{open} \text{ as } Ob\mathcal{OC} - \mathcal{D}^d_{open} \text{ bimod}$$



$$dD(\lambda_0, \dots, \lambda_{n-1}) = \sum_{\substack{0 \le i \le j \le n-1 \\ j-i \ge 2}} \pm D(\lambda_0, \dots, \lambda_j) * D(\lambda_j, \dots, \lambda_i)$$

glue Naive sense.

Nodal { can't determine what they are). could be open / category or incoming

(Lemma) ① $\mathrm{ob}\,\mathcal{OC}^d$ – $\mathcal{D}^d_{open}$ bimodule $\mathcal{D}^d$ is $\mathcal{D}^d_{open}$ - flat

② If M is a h-split $\mathcal{D}^d_{open}$ module, then $\mathcal{D}^d \otimes_{\mathcal{D}^d_{open}} M$ is h-split $\mathrm{ob}\,\mathcal{OC}^d$ – mod

(pf) idea ; $M \to M'$ left $\mathcal{D}^d_{open}$ module. supp. q.i

$$\mathcal{D}^d \otimes_{\mathcal{D}^d_{open}} M \longrightarrow \mathcal{D}^d \otimes_{\mathcal{D}^d_{open}} M' \text{ is quasi-iso.}$$

* filtration on generator of $\mathcal{D}^d$. $\partial\delta \in T^0$. $A(\lambda_0 \dots \lambda_{n-1}) \in F^n$

fact $dA(\lambda_0 \dots \lambda_{n-1})$ is F^{n-1}.

If we choose $\beta \in \mathrm{ob}\,\mathcal{OC}$

ETS. $\mathcal{D}^d(-, \beta) \otimes_{\mathcal{D}^d_{open}} M \longrightarrow \mathcal{D}^d(-, \beta) \otimes_{\mathcal{D}^d_{open}} M$.

associated graded guy is quasi-iso!

ex) $\beta = C \amalg \alpha$ when $c = 1$

This is quite trivial, by writing it down

(Step 3). ① $\Phi : \mathcal{D}^{+}_{open,\Lambda} \to Comp_{\mathbb{K}}$ is the same as a unital A_∞ cat. w/ some D-brane.

split

$$\Phi(D, s, t) \cong \bigotimes_{i=0}^{D-1} \Phi(\{s(i), t(i)\}).$$

if I write down $Hom(\lambda, \lambda') = \Phi(\{\lambda, \lambda'\})$. clear

$\mathcal{D}^{+}_{open,\lambda}$ generated by the discs $D^{+}(\lambda_0, \dots, \lambda_{n-1})$.

$\Rightarrow$ it gives a map

$m_{n-1} : Hom(\lambda_0, \lambda_1) \otimes \cdots \otimes Hom(\lambda_{n-2}, \lambda_{n-1}) \to Hom(\lambda_0, \lambda_{n-1})$.

of deg $n-3$

dD formula $\Rightarrow$ A_∞-relation.

ex $n = 2, 1$. $D^{+}(\lambda)$. $D^{+}(\lambda, \lambda)$ deg 0.

$\mathbb{K} \to Hom(\lambda, \lambda)$ id

② Split $\Phi : \mathcal{D}^{d}_{open,\Lambda} \to Comp_{\mathbb{K}}$. unital CY A_∞-cat

two incoming & two outgoing boundaries

$Hom(\lambda_0, \lambda_1) \otimes Hom(\lambda_1, \lambda_0) \to \mathbb{K}[d]$. and

its inverse.

extra condition $\Rightarrow$ cyclic symmetric

(Def) n-split sym mono. $\Phi : \mathcal{D}^{d}_{open,\Lambda} \to Comp_{\mathbb{K}}$.

(part 1). q. equiv. def 4.2.1 ($\because$ quasi-isomorphic)

(prop) (skip).

For part (2).

$Ob\, OC_\Lambda^d - \mathcal{D}^d_{open, \Lambda}$ bimodule of $\mathcal{D}_\Lambda^d \Rightarrow$ flat $\mathcal{D}^d_{open, \Lambda}$ - module.

$Ob\, OC_\Lambda^d - \mathcal{O}_\Lambda^d$ bimodule of OC_Λ^d.

So, let M : left $\mathcal{D}^d_{open, \Lambda}$ - module

M' : " $\mathcal{O}_\Lambda^d$ - module

$$\Rightarrow \underbrace{OC_\Lambda^d(-, \beta) \otimes^{L}_{\mathcal{O}_\Lambda^d} M'}_{N(\beta)} \cong \mathcal{D}_\Lambda^d(-, \beta) \otimes_{\mathcal{D}_{open, \Lambda}} M.$$

$\Rightarrow$ $N(\beta)$ is h-split if M is.

Thus it defines open-closed TCFT. " homotopically universal.

For part (3)

Φ unital extended CY A_∞ - Cat.

$$H_*(\mathcal{D}^d(-, 1)_\Lambda \otimes_{\mathcal{D}^d_{open}} \Phi) = HH_*(\Phi).$$

(idea) $\mathcal{D}^d(-, 1)_\Lambda \otimes_{\mathcal{D}^d_{open, \Lambda}} \Phi = \mathcal{D}^+(-, 1)_\Lambda \otimes_{\mathcal{D}^+_{open, \Lambda}} \Phi$.

(Fact). A_∞-cat $\Leftrightarrow$ dg Cat

So for dg cat B considered it as a left $\mathcal{D}^+_{open, \Lambda}$ - module

$$H_*(\mathcal{D}^+(-, 1)_\Lambda \otimes_{\mathcal{D}^+_{open, \Lambda}} B) = HH_*(B)$$

$\cong$

$\overline{C}_*(B)$ normalized Hochschild chain

A: dg cat $C_*(A) = \bigoplus_n (Hom(\alpha_0, \alpha_1) \otimes \cdots \otimes Hom(\alpha_{n-1}, \alpha_n))[1-n]$

$d(\phi_0 \otimes \cdots \otimes \phi_{n-1}) \cdots$ / $(\phi_0 \otimes \cdots \otimes \phi_{n-1})$ subcomplex spanned by at least one of the ϕ_i, where $i > 0$. is an identity map.