Talk.

- Preview: Want to construct Segal-type TFT with target category from derived (symplectic) algebraic geometry. [illegible]

- Preliminaries: PTVV. n-shifted symplectic str.
- Short proof of Main thm.
- Application / Conjecture

① Y : dArSt. n-shifted Symplectic.

$$\mathrm{Cob}_d^{\text{or.(fr)}} \longrightarrow \mathcal{C} = \mathrm{Lagcorr}.$$

$$Z \longmapsto \mathrm{Map}((Z)_B, Y). : \text{d-ArSt stack w/ } n-d+1 \text{ shifted Symplectic str.}$$

$$\begin{array}{c} X \\ \nearrow \quad \nwarrow \\ Z_1 \qquad\quad Z_2 \end{array} \longmapsto \mathrm{Map}(X_B, Y) \to \mathrm{Map}(Z_{1B}, Y) \times \mathrm{Map}(Z_{2B}, Y).$$

↑ Induction map.

[illegible] to Lagrangian [illegible]

- My talk is mostly focused on construction of this functor,
  Mostly, ⊛ Lagrangian correspondence!
- Main Application: $Y = BG$, or similar type by Moore, Tachikawa

② Preliminaries

- [illegible] sheafs

X : top'l space and $X_B(S) = C^*_{\mathrm{sing}}(X : \mathbb{k})$

$\Gamma(X_B, \mathcal{O}_{X_B}) = C^*_{\mathrm{sing}}(X, k)$.

$\mathrm{Qcoh}(X_B) \sim$ $(\infty, 1)$ Cat of locally const. sheaves of k-mod. on X

$\Gamma(X_B, \mathcal{F}) = \Gamma(X, \mathcal{F}) \quad \forall\, \mathcal{F} \in \mathrm{Qcoh}(X)$

(thm) $L_1 \xrightarrow{f_1} (X,\omega) \xleftarrow{f_2} L_2$ $\Rightarrow \exists$ $\mathbb{L}ag(f_1,\omega) \times \mathbb{L}ag(f_2,\omega) \to Sym(L_1 \overset{h}{\underset{X}{\times}} L_2, n-1)$

(i.e. $L_1 \times_{(X,\omega)} L_2$ has $(n-1)$ symplectic str.).

$\forall\ L \to X$, $A^{p.cl}(X/L, \omega) \to A^{p.cl}(X) \to A^{p.cl}(L)$.

$X/L = $ $\begin{matrix} L & \to & X \\ \downarrow & & \downarrow \\ 0 & \to & X/L \end{matrix}$. L Lag $\Rightarrow$ we can lift ω to $\tilde{\omega}$

$L \times_X L$: stack of path from L to L

$\sim$ stack of pointed loop X/L.

$\sim$ $Map(S^1, X/L)$ $\rightsquigarrow$ $n-1$ str

(Def) $f : \Upsilon \to \Sigma$ mor. bet $\mathcal{O}$-compact stacks.

$[\Upsilon]$ equipped w/ fundamental class

· Boundary str. on f is path from $f_*[\Upsilon]$ to 0.

In $Map(\Gamma(\Sigma, \mathcal{O}_\Sigma), k[-d])$

· $\mathbb{B}nd(f, [\Upsilon])$: space.

· Non-degenerate means ~~that~~ "relative PD-condition"

· A relative d-orientation on $f : \Upsilon \to \Sigma$ is

d-ori + non-degenerate boundary str.

(e.g.) $\partial M \hookrightarrow M$ ($\partial M = \Upsilon$, $M = \Sigma$)

$[M] \in H_{d+1}(M, \partial M, k) \longrightarrow H_d(\partial M, k) \longrightarrow H_d(M, k)$

$[M] \longmapsto [\partial M] = [\Upsilon] \longmapsto f_*[\Upsilon]$

$\therefore$ $[M]$ determines $f_*[\Upsilon] \sim 0$ in $H_d(M, k)$)

if $X = *_k \Rightarrow$ obj $\Rightarrow$ k-1 symplectic d st.

Conjecture: Any obj Y in $\mathrm{Lag}_{(m,n)}(*_{(k)})$ fully dualizable

(i.e.). $\mathrm{Bord}_n^{fr} \longrightarrow \mathrm{Lag}_{(m,n)}(*_k)$. given by $\mathrm{Map}((-)_B, Y)$

$$\mathrm{Map}((pt)_B, Y) = Y$$

Also if factors through oriented versions.

? No evidence!

③ Applications.

· Moore - Tachikawa conjecturally constructed 2d TFT whose target category is coming from symplectic holomorphic mfd.

$$\mathrm{Cob}_2 \longrightarrow \mathcal{E}$$

obj : Lie $gp/\mathbb{C}$ (alg - gp).

$T^*X[n]$

$= \mathbb{R}\mathrm{Spec}\,\mathrm{Sym}_{\mathcal{O}_X}(T_X[-n])$

mor : holo symplectic manifold. w/

$X \longrightarrow (\mathfrak{g}_1 \oplus \mathfrak{g}_2)^*$ moment map

i.e. $G \curvearrowright X$ w/ hamiltonian action

$id = T^*G_{\mathbb{C}}$

In our case, this setting can be interpreted as follows

$\mathcal{E} \longrightarrow \mathrm{Lag}_1$ $\mathbb{C}^*$ action

$G \longmapsto [\mathfrak{g}^*/G] = T^*[1]BG$. 1-symplectic

$X \qquad [X/G_1 \times G_2]$

$\downarrow \qquad \downarrow$ Lag composition. fiber product.

$(\mathfrak{g}_1 \oplus \mathfrak{g}_2)^* \qquad [\mathfrak{g}_1^*/G_1] \times [\mathfrak{g}_2^*/G_2]$

(Rmk). $X \to \mathfrak{g}^*$ moment map $\Rightarrow [X/G] \to [\mathfrak{g}^*/G]$ Lag.

pull back $\Rightarrow \mu^{-1}(0)/G$.