Worksheet 1

Remember, no credit will be given for answers without mathematical or logical justification.

Part I

1) Note that $439\,068 = 12 \times 36\,589$. If it is 7 o'clock right now, what time will it be in $439\,069$ hours?

For problems 2 and 3, we make the following definition. A number p is *prime* in mod n arithmetic if the only way that

$$p \equiv a \times b \ (mod\, n) \tag{1}$$

is that either $a \equiv 1 \ (mod\, n)$ and $b \equiv p \ (mod\, n)$, or $a \equiv p \ (mod\, n)$ and $b \equiv 1 \ (mod\, n)$.

2) In arithmetic mod 7, are there any primes? For instance, $3 \equiv 2 \times 5 \ (mod\, 7)$ so 3 is not prime.

3) Can you prove that, in mod 6 arithmetic, 5 is prime? Can you show that both 1 and 2 are *not* prime?

Part II

4) Consider the M-sequence (M is for Math 170), given by $M_1 = 1$, $M_2 = 1$ and for any $n > 2$

$$M_n = 2M_{n-1} + 3M_{n-2}. \tag{2}$$

List the first 8 M-numbers.

5) Consider the n^{th} ratio of M-numbers, defined to be

$$R_n = \frac{M_{n+1}}{M_n}.$$

Determine a recursive formula for the R_n.

6) Obviously the M numbers are getting big fast. But how fast? Determine, in the limit as n gets bigger, what the ratio R_n of successive M-numbers is.

Part III

7) A *dyadic rational* is a fraction whose denominator is a power of two. In other words, a fraction of the form

$$\frac{m}{2^n} \tag{3}$$

where m and n are integers. Which of the following numbers are dyadic rationals?

$$\pi, \quad \frac{1}{2}, \quad \sqrt{2}, \quad \frac{3}{8}, \quad \frac{8}{3}, \quad \frac{17}{16}, \quad 9, \quad \frac{1}{48} \tag{4}$$

8) Construct a one-to-one correspondence between the positive dyadic rationals and the natural numbers. Make a list of the first 13 dyadic numbers, according to your correspondence. (As in class, you may use a picture to describe your method, as long as it is clear.)

9) There are obviously many decimal numbers that are *not* on the following list of five numbers:

$$\begin{array}{l} .123\,456\,7 \\ .704\,515\,73 \\ .652\,843\,5 \\ .999\,999\,999\,92 \\ .111\,111\,111\,14 \end{array} \tag{5}$$

but for this problem, use the diagonal trick to find a number that is not on the list.

10) Do #13 of section 3.3 in the book.

11) Some of the mathematics we are discussing is both very ancient, and common to even the oldest city-building cultures—indeed mathematical systems appear to coincide with the rise of civilization itself. I mentioned several possible reasons for this: the need for equitable arbitration and dispute settlement, fair apportionment of taxes, measurements in the conduct of trade, management of surpluses, and for aspects of military efficiency. In your view, do you think the development of mathematics precedes civilization and then participates in its rise? Or do you think civilization came first, and then mathematics was developed to help solve its organizational problems? Or do you think something else is the case—maybe you have an example of an ancient civilization without mathematics?

    Obviously we don't *know* the answers to this question (although archaeology may shed light on it). But I want you to make your most reasonable conjecture, and justify it with a sound argument or two. Write about a paragraph or so.