MATH 114 ASSIGNMENT 1, FALL 2015

Due in class on Friday, September 4th

Part 1. Read Thomas 12.1 to 12.5 and make sure you can do all core problems in these three sections.

Part 2. Do and write up the following problems from Thomas:

12.1 Exercises 56, 66

12.2 Exercises 43, 48

12.3 Exercises 23, 24, 42, 46

12.4 Exercises 34, 44, 50

12.5 Exercises 52, 58

Part 3. Extra credit problem.
E. The *Hamiltonian quaterions* consists of all formal expressions of the form

$$u + x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k}, \qquad u, x, y, z \in \mathbb{R}.$$

The addition of two Hamiltonian quaternions are defined cooridinate-by-coordinate:

$$u_1 + x_1 \cdot \vec{i} + y_1 \cdot \vec{j} + z_1 \cdot \vec{k} + u_2 + x_2 \cdot \vec{i} + y_2 \cdot \vec{j} + z_2 \cdot \vec{k} = (u_1 + u_2) + (x_1 + x_2) \cdot \vec{i} + (y_1 + y_2) \cdot \vec{j} + (z_1 + z_2) \cdot \vec{k}$$

E1. Define multiplication of quaternions so that the following hold:

(a) real numbers commute with $\vec{i}, \vec{j}, \vec{k}$ under multiplication

(b) associativity rule holds for multiplication

(c) the two distributive rules hold

(d) $\vec{i} \cdot \vec{j} = \vec{k} = -\vec{j} \cdot \vec{i}, \quad \vec{j} \cdot \vec{k} = \vec{i} = -\vec{k} \cdot \vec{j}, \quad \vec{k} \cdot \vec{i} = \vec{j} = -\vec{i} \cdot \vec{k}.$

E2. Use the arithmetic of Hamiltonian quaternions to "explain" the non-associativity of the cross product. (This way the cross product is seen as part of an operation with "better" properties.)