MATH 114 ASSIGNMENT 10, FALL 2015

Due in class on Friday, November 13. (You will see Xuran November 13.)

We have defined oriented surface integrals and oriented line integrals on November 4th, and stated Stokes' and the divergence theorem on November 6th. These theorems are the heart of multivariable calculus. They are the fundamental theorem of calculus in dimensions 1&2 and 2&3 respectively; you can think of them as "integration by parts in higher dimensions". (The stuff of independence of path for line integrals, explained in 16.3 of Thomas, will be explained later.)

It will take quite some time, with the help with many examples, to understand the fundamental theorem of calculus. Most of the lectures after November 6th will be devoted to examples and illustrations of the fundamental theorem of calculus.

This set of assignments is about the *definition* of oriented surface integrals and oriented line integrals. Orientation an important ingredient here; assignment 9 is devoted to orientations.

Many of the problems of section 16.6 in Thomas ask you to compute the flux of a vector field, i.e. the oriented surface integral of the given vector field. In these problems the surface is connected, and you are given some indications about which of the two possible orientation is chosen. Figuring out which of two possible orientations of the surface is used and determining the *sign* of the parametrization with respect to the chosen orientation are two important part of these questions.

Part 1. Read 16.2 and 6.6 of Thomas, plus §3 of the notes for vector calculus.
Note: You can treat the last part of 16.2 of Thomas, on the *flux* of a vector field $\vec{F} = M(x,y)\,\vec{i} + N(x,y)\,\vec{j}$ on the plane $\mathbb{R}^2$, over an oriented closed curve $(C,\vec{T})$ on the plane, as a definition: it is

$$\oint_{(C,\vec{T})} M\,dy - N\,dx.$$

Suppose that $(C,\vec{T})$ is the oriented boundary of a bounded region R in the (x,y)-plane and $\vec{F}$ has *no singularity in* R, where it is understood that R is oriented by the constant vector field $\vec{k}$, this flux is equal to

$$\iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy.$$

by Stokes' theorem (or Green's theorem).

Part 2. Do and write up the following problems, from Thomas and old final exams.

16.2 Exercise 38 (a), (b).

16.6 Exercise 38. [Here the unit normal vector field $\vec{N}$ for the surface has the property that its projection to the (x,y)-plane is a *positive* multiple of the vector field $x\vec{i} + y\,\vec{j}$.]

16.6 Exercise 40. [In the sentence specifying the orientation of the surface, i.e. the direction of the unit normal vector field $\vec{N}$, the meaning of the phrase "pointing away from the (x,z)-plane" is unclear. Here we will interpret it as: the k-component of $\vec{N}$ is positive.]

S14 Math 114 final exam, May 5, 2014, problem 12. [Here the surface is the part of the boundary of the half-space $H := \{(x,y,z) \in \mathbb{R}^3 \,|\, 3x + y + 2z - 6 \leq 0\}$, which contains the origin $(0,0,0)$. The unit normal vector field $\vec{N}$ chosen for the surface S is the one which points away from the half-space H.]

S14 Math 114 final exam, May 5, 2014, problem 13. [Here the surface is the boundary ∂D of the solid D, and ∂D is oriented by the unit normal vector field on ∂D which points away from D—on smooth points of ∂D.]

S13 Math 114 final exam, December 17, 2013, problem 13. [Here **n** is the unit normal vector field on the unit sphere which points away from the solid unit sphere. In other words

$$\mathbf{n}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$$

for all points (x,y,z) satisfying $x^2+y^2+z^2=1$.]