

MATH 114 ASSIGNMENT 12, FALL 2015

Due in class on Monday, November 30.

Part 1. Do and write up the following problems.

240S06 Math 240 final exam, December 14, 2006, problem 7.

240S07 Math 240 final exam, December 12, 2007, problem 13.

Part 2. Extra credit problems.

A. (change of variables for oriented surface integrals) Let $\vec{F} = \frac{1}{(x^2+y^2+z^2)^{3/2}} (x\vec{i} + y\vec{j} + z\vec{k})$. Find a vector field G in the (ρ, ϕ, θ) -space such that for any oriented surface (S_1, \vec{N}_1) in the (ρ, ϕ, θ) -space given by a parametrization $\vec{r}_1 : D \rightarrow \mathbb{R}_{\rho, \phi, \theta}^3$, we have

$$\iint_{(S_2, \vec{N}_2)} \vec{F} \cdot \vec{N}_2 d\sigma_{x,y,z} = \iint_{(S_1, \vec{N}_1)} \vec{G} \cdot \vec{N}_1 d\sigma_{\rho, \phi, \theta}$$

where (S_2, \vec{N}_2) is the oriented surface in the (x, y, z) -space given by the parametrization $\vec{r}_2 = \Phi(\vec{r}_1)$, and Φ is the map from the space with coordinates (ρ, ϕ, θ) to the space with coordinates (x, y, z) , given by the usual formula in spherical coordinates

$$\Phi : (\rho, \phi, \theta) \mapsto (\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi).$$